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there for all to behold; some as serene as da Vinci’s ‘Madonna of the Rocks’, others as powerful and majestic as Michelangelo’s glorious ceiling of the Sistine Chapel. In this sense, its own beauty is to the Copenhagen Interpretation which claims that ‘objective reality has evaporated’ and ‘quantum mechanics does not represent particles, but rather our knowledge, our observations, or our consciousness, of particles”. Popper points out that, over the years, many eminent physicists have switched allegiance from the pro-Copenhagen view. In some ways, the most important of these people was David Bohm, a greatly respected thinker on scientific matters who wrote a book presenting the Copenhagen view of quantum mechanics in minute detail. However, later, apparently under Einstein’s influence, he reached the conclusion that his previous view had been in error and also declared the total falsity of the constantly repeated dogma that the quantum theory is complete. It was, of course, this very question of whether or not quantum mechanics is complete which formed the basis of the disagreement between Einstein and Bohr; Einstein stating “No”, Bohr “Yes”.

Although ‘conventional wisdom’ dictates otherwise, both the widely accepted theories of relativity and quantum mechanics, particularly quantum mechanics, are incomplete. The quarks surrounding both had been hinted at but possibly more has emerged concerning the inadequacies of quantum mechanics because of the progress being made. Notably, although it is not publicly stated too frequently, Einstein had great doubts about various aspects of quantum mechanics. Much of the worry has revolved around the role of the observer and over the question of whether quantum mechanics is an objective theory or not. One notable contributor to the debate has been that eminent philosopher of science, Karl Popper. As discussed in his book, “Exploding a Myth”, Popper preferred to refer to the experimentalist rather than observer, and expressed the view that that person played the same role in quantum mechanics as in classical mechanics. He felt, therefore, that such a person was there to test the theory. This is totally opposed to the Copenhagen Interpretation which claims that ‘objective reality has evaporated” and “quantum mechanics does not represent particles, but rather our knowledge, our observations, or our consciousness, of particles”.

Foreword

Mathematics is a subject which possibly finds itself in a unique position in academia in that it is viewed as both an Art and a Science. Indeed, in different universities, graduates in mathematics may receive Bachelor Degrees in Arts or Sciences. This probably reflects the dual nature of the subject. On the one hand, it may be studied as a subject in its own right. In this sense, its own beauty is found in the real world; if results predict some effect, that prediction must be verified by observation and/or experiment. Again, it may be remembered that mathematics as the language of science, of physics in particular since physics is mathematics as the language of science, of physics in particular since physics is an Art and a Science. Indeed, in different areas of science at the very hub of all scientific endeavour, all other branches are incomplete. The qualms surrounding both have been muted but possibly more 

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Contrary to a rather popular belief, quantum mechanics is only approximately valid for a number of particle conditions at short mutual distances. A clear example is the Bose-Einstein correlation in which protons and antiprotons collide at very high energy, annihilate each other, and result in the production of a large number of mesons that remain correlated at large mutual distances. On strict scientific grounds, a theory constructed for the orbiting of point-like electrons in vacuum around atomic nuclei is not expected to be exactly valid for the dramatical conditions when it allows a numerically exact representation of all experimental data from submillifar quantum events, such as the electron (called "elementary particles") to provide the missing energy, has no credibility because the hypothetical antineutrino has an absolutely null cross section with protons and electrons. A similar basic inapplicability of quantum mechanics occurs for numerous other cases whose treatment is generally ignored or claimed as not needed, such as the synthesis of the neutron from protons and electrons as occurring in stars, or, more generally, the synthesis of strongly interacting particles (called "hadrons") require a physical solution for positive binding energies, as the skeptic reader is encouraged to verify. The attempt of solving quantum mechanics via the conjectural reaction $p^+ + e^- \rightarrow n + \nu$, namely, the dream of using the hypothetical antineutrino to provide the missing energy, has no credibility because the hypothetical antineutrino has an absolutely null cross section with protons and electrons. A similar basic inapplicability of quantum mechanics occurs for numerous other cases whose treatment is generally ignored or claimed as not needed, such as the synthesis of the neutron via the known reaction $e^+ + e^- \rightarrow \gamma$, as well as for the synthesis of all exotic particles.

The author has dedicated his research life to the study of the limitations of conventional theories, the construction of suitable generalization, and their application to the industrial development of clean and remote energy sources. The studies initiated with paper [1] of 1956 (written when the author was an undergraduate student of physics at the University of Napoli, Italy), on the conception of space, or, vacuum as a universal medium (or substratum) of high density and energy. The paper was written for the resolution of the controversy on the "etheral wind" raging at that time via the reduction of all particles constituting matter, such as the electron, to "pure oscillations of space," namely, oscillations of the space itself without any oscillating conventional mass.

Under these conditions, when masses are moved, there cannot be any etheral wind since we merely move oscillations of space from given points to others [1]. According to this view and in dramatic contrast with our sensory perception, matter is completely empty in the sense that it can be entirely reduced to pure oscillations of space without any oscillation of conventional masses, as necessarily necessary for the structure of the electron. Consequently, the viewer requires that space is completely full of a medium of extremely high density (from the very large value of the speed of light) and, thus, space was conceived in Ref. [5] as possessing a feature approximating our notion of rigidity from the purely transcendental character of light.

The study of space as a universal medium is significant for the main objectives of these volumes, including the search for new clean energies. In cosmology, we have the long standing hypothesis of the continuous creation of matter in our universe. In the event this hypothesis is correct, the most plausible origin of the matter of the universe, is that part of the whole possessing the beauty so beloved of mathematicians and great artists. However, the scientific community should reserve its final judgement until it has had a chance to view the experimental and practical evidence which may be produced later in support of this elegant new theoretical framework.

Our planet is affected by increasingly catastrophic climatic changes. The only possibility for their containment is the development of new, clean, energetic and safe fuels. But, all possible energies and fuels that could be conceived with quantum mechanics, quantum chemistry, special relativity, and other conventional theories, had been discovered by the middle of the 20-th century, and they all resulted in being environmentally unacceptable either because of an excessive production of atmospheric pollutants, or because of the release of dangerous waste.

The scientific community of the 21-st century is faced with the quite complex situation of both broadening conventional theories into forms permitting the prediction and quantitative study of new clean energies and fuels and, secondly, developing them up to the needed industrial maturity. These volumes outline the efforts conducted by the author and a number of other scientists, as well as3 institutional toward these pressing needs of the human society.

To begin, we shall say that a theory is: 1) exactly valid for given physical conditions when it allows a numerically exact representation of all experimental data from submillifar quantum events, 2) approximately valid for different physical conditions when requiring the use of unknown parameters to fit the experimental data, and 3) basically inapplicable for yet different conditions when unable to provide any quantitative treatment even with the use of arbitrary parameters. Note that quantum mechanics, quantum chemistry, special relativity and other theories of the 20-th century, cannot be claimed to be "violated" for conditions 2) and 3) since, as we shall see, they were not conceived for the latter conditions. There is no doubt that quantum mechanics permitted in the 20-th century the achievement of historical advances in various fields. These successes caused a widespread belief that quantum mechanics is exactly valid for all possible conditions of particles existing in the universe. Such a belief is scientifucally, particularly when ventrusted by experts. As established by history, sciences will never admit final theories. No matter how valid any theory may appear at a given time, its structural generalization for a representation of previously unknown conditions is only a matter of time.

In conclusion, quantum mechanics is exactly valid for the physical conditions of its original conception, point-like particles and electromagnetic waves propagating in vacuum, as occurring in the structure of the hydrogen atom, the structure of crystals, the motion of particles in an accelerator, and numerous other conditions.
the representation of the binding energy via the screened Coulomb law is merely approximately valid and cannot be credibly claimed to be exactly valid for molecular structures even after the screening of the Coulomb law. A

an attractive force, of an exact representation of the binding energies and other unadulterated Coulomb law feature. By comparison, despite its widespread use generally without a serious
century is a pure nomenclature deprived of quantitative content.

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requirements: i) the precise identification of the origin of the bonding force; ii) the achievement, with such attempts, with bigger deviations for the water molecule

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were proposed for the treatment of open irreversible events (such as energy re-

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The contact nonpotential character of the deep mutual penetration of the two

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new mathematics beginning with the original proposal to construct hadronic isodual special relativity as an evident necessary condition to achieve compatibility with the operator treatment. This identifies the need for numbers, spaces, differential calculus, solely applicable on a Hilbert space over the field of complex numbers. Hence, homomorphic or, more generally, anti-isomorphic to conventional mathematics although only for certain states called "isoselfdual" [16]; the prediction that light oering for the first time in history the possibility in due time of ascertaining the new basic numbers.

The author has stated several times in his papers that special relativity has a "magnetic automatic structure and validity" for the original proposal to construct hadronic isodual special relativity as an evident necessary condition to achieve compatibility with the operator treatment. This identifies the need for numbers, spaces, differential calculus, solely applicable on a Hilbert space over the field of complex numbers. Hence, homomorphic or, more generally, anti-isomorphic to conventional mathematics although only for certain states called "isoselfdual" [16]; the prediction that light oering for the first time in history the possibility in due time of ascertaining the new basic numbers.

The second illustration of the need for new mathematics is given by the relativity principle articulated by Einstein, namely, for point-like particles and electromagnetic forces propagating in vacua, such as for the structure of the hydrogen atom, particles moving in accelerators, etc. However, for numerous different conditions, special relativity is either approximately valid or basically inapplicable. There are numerous conditions for which special relativity is basically inapplicable (rather than violated, because not conceived for the conditions at hand). For instance, special relativity is inapplicable for the classical treatment of antimatter, as clearly established by the absence of any differentiation between neutral matter and antimatter. Special relativity is also inapplicable for the classical representation of charged antiparticles because, in view of the existence of only one quantization channel, the operator image of a classical antiparticle is that of a classical particle in a single channel using a charge sign of the change. At any rate, antimatter had not yet been conceived, let alone detected, at the time of the inception of special relativity. Hence, the current widespread use of special relativity for the classical description of antimatter is a scientific manipulation by Einstein's followers, and definitely not a scientific blunder by Albert Einstein.

Similarly, special relativity is inapplicable for a quantitative treatment of the chemical valence or, along similar lines, for the contact, nodal and nonpotential conditions of deep inelastic scatterings of particles, because its mathematical structure simply cannot represent forces not admitting a Hamiltonian representation by. When passing to the main scope of these volumes, energy releasing processes, their study via special relativity is outside the boundaries of science. This is due to the fact that all energy releasing processes are structurally irreversible in time, in the sense of being irreversibly for all possible Hamiltonians, while special relativity is known to be structurally reversible for all known Hamiltonians are reversible in time. It is evident that a theory proved to be valid for the representation of the time reversal invariant orbits of atomic electrons, cannot permit a serious scientific study of irreversible energy releasing processes. As an example, a special relativity predicts that light travels beyond the boundary of a black hole, which is the reason for the great effectiveness of quantum mechanics for the treatment of atomic electrons, cannot permit a serious scientific study of irreversible energy releasing processes. As an example, a special relativity predicts that light travels beyond the boundary of a black hole, which is the reason for the great effectiveness of quantum mechanics for the treatment of atomic electrons, cannot permit a serious scientific study of irreversible energy releasing processes.

Following additional trials and errors, the new mathematics was constructed beginning with the original proposal of 1978 to construct hadronic mechanics [4,5] and then continuing in numerous works (see the mathematical presentation [19] of Santilli's nonpotential isodual isomathematics for the treatment of antimatter. The new numbers were constructed for the first time in paper [14] of 1982. The author discovered in this way that contemporary treatments of the unit 1 dating back to biblical times. In fact, as we shall see, the assumption of the value 1/3 as unit implies that 2 x 3 = 18 and "4 a prime number".

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The most complex efforts dealt with an irreversible generalization of special relativity into a form, today known as Santilli hyper-relativity admitting simultaneity as well as the conventional special relativity as particular cases, with corresponding isoduality for antimatter. A main difficulty was given by the need to achieve structurally irreversible symmetries characterizing time rates of variation of physical quantities, as occurring in nature. The solution was permitted by the Lie-admissible covering of Lie theory using studies initiated in 1967 [2] (see also [19,20]).

The indicated lack of final theories in science was confirmed by the fact that all the preceding or different mathematical (conventional, isotopic and genotopic, and their isoduals) resulted in being inefficient for serious studies in biology since, for reasons we shall see, the latter require math-valued methods. This occurrence can be intuitively seen from the fact that, e.g., a few atoms in a DNA can generate a complex organism with a huge number of code. A multi-valued mathematics did exist in the此事oid, the so-called isofunctional structures, but they had no possibility of applications to biological structures due to the absence of a left and right unit (evidently crucial to permit measurements), the use of rather abstract operations not compatible with experiments, and other reasons.

These limitations led the author to the construction of a final form of mathematics, today known as Santilli hyper-relativistic that is irreversible, multi-valued and possesses a left and right unit at all levels. Santilli's hyper-relativistic is then the corresponding form for antimatter [2].

After the above historico-research, including the construction of the above new mathematics and related broadening (called lifting) of quantum mechanics, quantum chemistry, special and general relativities, the author had still failed to achieve by the early 1990s a property truly crucial for serious physical value, the issuance of the numerical predictions under the time evolution of the theory, namely, the prediction of the same numerical values under the same conditions at different times. The indicated new mathematics did indeed provide a sequence of generalizations of Hamilton's classical equations, Heisenberg's operator equations, Einstein's equasion for the special relativty, etc., but their numerical predictions under the same physical conditions turned out to change over time, a catastrophic inconsistency that delayed the applications of hadronic mechanics for decades because the author simply refused to publish papers he considered catastrophically inconsistent.

Again, major physical problems generally originate from insufficient mathe- matics, and the solution emerged from the identification of another one of the other popular belief in pure mathematics, the belief that the differential calculus does not depend on the basic fact (the dimension of another real number). The mathematical belief is correct only for constant units, since said belief is no longer valid whenever the generalised units depend on the local coordinates. This occurrence permit-

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Following numerous years of research, a resolution of these inconsistencies was reached via a geometric unification of special and general relativities beginning with a geometric unification of the Minkowskian and Riemannian geometries [23] via the abstract Minkowskian aknowledge thanks to the power of isometrics. According to this view, any Riemannian metric \( g(x) \) is decomposed into the Minkowskian metric \( I(x) \) and the metric \( g(x) = I(x) + \lambda \) with isometric \( \lambda(x) \) but flat, such as \( \lambda(x) \) being given by the universe of the gravitational matrix, \( I(x) = 1 + \lambda(x) \). This procedure eliminates the origin of all problems of Einstein's gravitation, curved space, since \( I(x) \) is flat (this formulation of gravity was presented by the author at the Marcel Grossman Meeting in Gravitation of 1994 [24]).

The new conception of gravitation without curvature permitted, apparently for the first time, the resolution of one century old controversies on Einstein's gravitation as well as: to achieve a universal generalization of the conventional and isotopic elements, the Poincare-Santilli isospacity [5], the achievement of a fully consistent operator formulation of gravity, including a fully valid PCT theorem, via the embedding of gravitation in the set of relativistic momentum mechanics and an axiomatic consistent grand unification of electromechanical and gravitational interactions including, for the first time, matter and antimatter, and based on the universal Poincaré-Santilli isospacity (unification presented at the Marcel Grossman meeting in gravitation of 1996 [25]).

As we shall see, all the above atoms are most crucial one being the representation of gravity without curvature, suggest rather radical new vistas in cosmology, such as:

1) The possibility of experimental resolution in due time whether far away galaxies and quasars are made of matter or antimatter via the predicted gravitational repulsion caused by matter on light emitted by antimatter and other experimental means;

2) The most logical interpretation of the expansion of the universe permitted by matter and antimatter galaxies and quasars, since their gravitational repulsion allows a quantitative representation not only of the expansion of the universe but also increases in time;

3) Dramatic revisions in the notion of time that becomes local, i.e. varying from an astrophysical body to another and with opposite signs for matter and antimatter, with a possible "null total time of the universe" that would avoid immense discontinuities at creation, such as those implied in the "big bang";

4) The first known cosmology with a universal symmetry, the Poincare-Santilli isospacity for matter multiplied by its dual for antimatter and...
The author has repeatedly stated in his works that Albert Einstein is, unquestionably, the greatest scientist of the 20th century, but he is also the most exploited scientist in history to date, because a large number of researchers have exploited Einstein’s name for personal gains in money, prestige, and power.

In these two volumes, we shall honor Einstein’s name as much as scientifically possible, but we shall jointly express the strongest possible criticisms of some of Einstein’s followers, by presenting a plethora of views in which Einstein’s name has been abused for conditions dramatically beyond those conceived by Einstein, under which conditions his theories are inapplicable (rather than violated), be exploited Einstein’s name for personal gains in money, prestige, and power.

In so doing, Einstein’s followers have created one of the biggest scientific obituaries since, as technically shown in these volumes, the resolution of our current environmental problems requires new scientific vistas.

The author has repeatedly stated in his works that Albert Einstein is, unquestionably, the greatest scientist of the 20th century, but he is also the most exploited scientist in history to date, because a large number of researchers have exploited Einstein’s name for personal gains in money, prestige, and power.
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Acknowledgments

The author expresses his deepest appreciation in memory of: the late British philosopher Carl Popper, for his strong support in the construction of hadronic mechanics, as shown in the Preface of his last book Quantum Theory and the Quantum World; the late Nobel Laureate Ilya Prigogine, for pioneering the need of symmetry breaking of quantum theory and his personal support for the organization of the Hadronic Journal since inception; the late Italian physicist Pietro Calabria, for his pioneering work in noncanonical broadening of conventional canonical theories as well as support for the construction of hadronic mechanics, the Greek mathematician Grigorios Tsagas, for fundamental contributions in the Lie-isotopic methods underlying hadronic mechanics, the late Italian physicist Guido Fubini, for pioneering nonassociative departures from the geometric structure of special relativity, extended by hadronic mechanics into anastropic and anisomorphic media, the late mathematician Robert Ojha-Meliana, for pioneering work on the Lie-admissible structure of hadronic mechanics; the mathematician Jaak Lohmus whose studies on nonassociative algebras, with particular reference to the octonion algebra, have been particularly inspiring for the mathematician Jaak Lohmus whose studies on nonassociative algebras, with particular reference to the octonion algebra, have been particularly inspiring for the construction of hadronic mechanics; and other scholars who will be remembered by the author until the end of his life.

The author expresses his appreciation for invaluable comments to all participants of the International Workshop on Animistic Gravity and Anti-Hydrogen Atom Spectroscopy held in Sepino, Molise, Italy, in May 1996; the Conference of the International Association for Relativistic Dynamics, held in Washington, D.C., in June 2002; the International Congress of Mathematics, held in Hong Kong, in August 2002; the International Conference on Physical Interpretation of Relativity Theories, held in London, September 2002; and 2004; and the XVIII Workshop on Hadronic Mechanics held in Karlstad, Sweden, in June 2005. The author would like also to express his deepest appreciation to Professors A. van der Meer, Editor of Foundations of Physics; P. Vetro, Editor of Research in Chrono-Physics; M. Guzik, Editor of Hadronic Journal; V. A. Glikov, Editor of Journal of Modern Physics; S. Hosszú, Editor of Mathematical Methods in Applied Sciences; D. V. Alderbra, Editor of the International Journal of Modern Physics; T. N. Venikov, Editor of the International Journal of Hydrogen Energy; H. Feshbach, Editor of the MIT Journal of Physics, the Editors of the Italian, American, British, French, Russian, Italian and other physical and mathematical societies;

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In faith

Ruggero Maria Santilli

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Tarpon Springs, Florida, U. S. A.

October 11, 2007

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Chapter 1

SCIENTIFIC IMBALANCES OF THE TWENTIETH CENTURY

1.1 THE SCIENTIFIC IMBALANCE CAUSED BY ANTIMATTER

1.1.1 Needs for a Classical Theory of Antimatter

The first large scientific imbalances of the 20th century studied in the monograph is that caused by the treatment of matter at all possible levels, from Newtonian to quantum mechanics, while antimatter was solely treated at the level of second quantization [3].

Besides an evident lack of scientific democracy in the treatment of matter and antimatter, the lack of a consistent classical treatment of antimatter left open a number of fundamental problems, such as the inability to study whether a faraway galaxy or quasar is made up of matter or of antimatter, because such a study requires first a classical representation of the gravitational field of antimatter, as an evident pre-requisite for the quantum treatment (see Figure 1.1).

It should be indicated that classical studies of antimatter simply cannot be done by merely reversing the sign of the charge, because of inconsistencies due to the existence of only one quantization channel. In fact, the quantization of a classical antiparticle solely characterized by the reversed sign of the charge leads to a particle (rather than a charge conjugated antiparticle) with the wrong sign of the charge.

It then follows that the treatment of the gravitational field of suspected antimatter galaxies or quasars cannot be consistently done via the Riemannian geometry in which there is a simple change of the sign of the charge, as rather popularly done in the 20th century, because such a treatment would be structurally inconsistent with the quantum formulation.

At any rate, the most interesting astrophysical bodies that can be made up of antimatter are neutral. In this case general relativity and its underlying Riemannian geometry can provide no difference at all between matter and antimatter stars due to the null total charge. The need for a suitable new theory of antimatter then becomes beyond credible doubt.

As we shall see in Chapter 14, besides all the above insufficiencies, the biggest imbalance in the current treatment of antimatter occurs at the level of grand unifications, since all present attempts to achieve a grand unification of electromagnetic, weak and gravitational interactions are easily proved to be inconsistent under the request that the unification should hold not only for matter, as universally done until now, but also for antimatter. Hence, prior to venturing judgments on the need for a new theory of antimatter, serious scholars are suggested to inspect the entire scientific journey including the iso-grand-unification of Chapter 14.

1.1.2 The Mathematical Origin of the Imbalance

The origin of this scientific imbalance was not of physical nature, because it was due to the lack of a mathematics suitable for the classical treatment of antimatter in such a way as to be compatible with charge conjugation at the quantum level.

Charge conjugation is an anti-homomorphism. Therefore, a necessary condition for a mathematics to be suitable for the classical treatment of antimatter is that of being anti-homomorphic, or, better, anti-isomorphic to conventional mathematics.

Therefore, the classical treatment of antimatter requires number, fields, functional analysis, differential calculus, topology, geometries, algebras, groups, symmetries, etc. that are anti-isomorphic to their conventional formulations for matter.

The absence in the 20th century of such a mathematics is well established by the fact of a formulation of trigonometric, differential and other elementary functions, let alone complex topological structures, that are anti-isomorphic to the conventional ones.

In the early 1980s, due to the absence of the needed mathematics, the author was left with no other alternative than its construction along the general guidelines of hadronic mechanics, namely, the construction of the needed mathematics from the physical reality of antimatter, rather than adapting antimatter to pre-existing insufficient mathematics.

After considerable search, the needed new mathematics for antimatter resulted in being characterized by the most elementary and, therefore, most fundamental possible axioms, that of a negative unit

\[ -1 \]

1.1.3 Outline of the Studies on Antimatter

Recall that "science" requires a mathematical treatment producing numerical values that can be confirmed by experiments. Along these lines, Chapter 2 is devoted, first, to the presentation of the new mathematics suggested by the author for the classical treatment of antimatter under the name of isosymplectic analytic theory [94]. The first is, however, in continuous evolution, thus warranting an update.

Our study of antimatter initiates in Chapter 2 where we present the classical formalism, proposed under the name of isosymplectic analytic theory that begins with a necessary reformulation of Newton's equations and then turns to the needed analytic theory.

The operator formalism turned out to be equivalent, but not identical, to the quantum treatment of antiparticles, and was submitted under the name of isosymplectic quantum mechanics.

Following these necessary foundational studies, Chapter 2 includes the detailed verification that the new isosymplectic analytic formalism indeed verify all classical and particle experimental evidence.

In subsequent chapters we shall then study some of the predictions of the new isosymplectic analytic theory, such as antigravity, a causal time machine, the isosymplectic cosmology in which the universe has null total characteristics, and other predictions that are so far reaching as to be at the true edge of imagination.

All these aspects deal with point-like antiparticles. The study of extended, non-local and deformable antiparticles (such as the anti-proton and the antinucleon) initiates in Chapter 3 for reversible conditions and continues in the subsequent chapters for broader irreversible and multi-valued conditions.

1.2 THE SCIENTIFIC IMBALANCE CAUSED BY NONLOCAL-INTEGRAL INTERACTIONS

1.2.1 Foundations of the Imbalance

The second large scientific imbalance of the 20th century studied in this monograph is that caused by the reduction of contact non-local-integral interactions to a

\[ \text{contact non-local-integral interactions} \]

that such a number was assigned to the far left and right end of the mathematics to be presented. Fortunately, the first transparent and easy-to-read and for mathematicians who translated into a clear contact non-local-integral interactions to a

\[ \text{contact non-local-integral interactions} \]
interactions as well as contact nonpotential interactions.

In Sect. 1.1 and in Chapter 2, we show the structural imaliability of special relativity to permit a classical representation of antimatter in a form compatible with charge conjugation. In this section and in Chapter 3, we show the inability of special relativity to represent extended, nonpoint-like and deformable particles or antiparticles and/or their wavepackets under nonlocal-integral interactions at short distances.

In Sect. 1.3 and in Chapter 4, we show the irreconcilable inapplicability of special relativity for all possible, classical and operator irreversible systems of particles and antiparticles. The widely ignored theorem of catastrophic inconsistencies of Einstein's gravitation are studied in Sect. 1.4 and in Chapter 3.

A primary purpose of this monograph is to show that the political adaptation of everything existing in nature to special relativity, rather than constructing new relativity to properly represent nature, prevents the prediction and quantification of new clean energies and fuels so much needed by mankind.
of having an unrestricted functional dependence, including that on accelerations and other non-Newtonian forms. Thus, we shall see, nonselfadjoint forces generally non-Newtonian, in the sense of being non-selfadjoint, represent three-vectors; and the convention of the sum of repeated indices is hereon assumed.

Lagrangian and Hamiltonian equations, those derivable from a potential, or variationally selfadjoint (SA) forces, and those not derivable from a potential but are variationally nonselfadjoint (NSA) forces according to the above expression:

\[ H = \frac{p^2}{2m} + V(t,r,p), \quad \tag{1.2.1a} \]
\[ V = U(t,r,p) + \frac{1}{2} \sum \frac{d^2 U(t,r)}{d^2 p^i} p^i p^i, \quad \tag{1.2.1b} \]

where: \( p \) and \( v \) represent threecomponent vectors; and the convention of the sum of repeated indices is hereon assumed.

For this reason, Lagrange, Hamilton, and other founders of analytic dynamics generated their celebrated equations with external forces representing precisely the contact, zero-range, nonpotential forces among extraterrestrial particles. Therefore, the treatment of interior systems require the true Lagrange and Hamilton analytic equations, those with external terms:

\[ \frac{\delta H(t,r,v)}{\delta v^a} \frac{d}{dt} \frac{\delta H(t,v)}{\delta \dot{v}^a} - \frac{\delta L(t,r,v)}{\delta \dot{v}^a} = F^a(t,r,v), \quad \tag{1.2.2a} \]
\[ F(t,r,v) = F(t,r,v,m), \quad \tag{1.2.2b} \]

In particular, the reader should keep in mind that, while selfadjoint forces are of Newtonian type, nonselfadjoint forces are generally non-Newtonian, in the sense of being non-selfadjoint. Furthermore, the true Lagrange's and Hamilton's equations without external terms can only represent in the coordinates of the experimenter exterior dynamical systems, whereas the representations of interior dynamical systems in the given coordinates of the experimenter require the necessary use of the true analytic equations with external terms.

Whenever posed to dynamical systems not entirely representable via the sole knowledge of a Lagrangian or a Hamiltonian, a rather general attitude is that of transforming them into an equivalent purely Lagrangian or Hamiltonian form, these transformations are indeed mathematically possible, but they are physically insidious. It is known that, under sufficient continuity and regularity conditions and under the necessary reduction of nonconservative external terms to local approximations such as that in Eq. (1.2.4), the Darboux's theorem of the symplectic geometry or, equivalently, the Lie-Koerning theorem of analytic mechanics assure the existence of coordinate transformations of the form:

\[ \left\{ (t,r,v) \to (t',r',p'), \right\} \quad \tag{1.2.5} \]

under which nonselfadjoint systems (1.2.2) can be turned into selfadjoint form (1.2.1), thus eliminating the external terms.

However, coordinate transformations (1.2.5) are necessarily nonlinear. Consequently, the new reference frames are necessarily nonrotational. Therefore, the elimination of the external nonselfadjoint forces via coordinate transforms cause the necessary loss of Galileo's and Einstein's relativities.

Moreover, it is evidently impossible to place measuring apparatus in new coordinate systems of the type \( r' = \exp(\kappa p), \) where \( \kappa \) is a constant. For these reasons, the use of Darboux's theorem or of the Lie-Koerning theorem was strictly prohibited in monographs [9,10,11]. Thus, to avoid misrepresentations, the following basic assumption is hereon adopted:

**ASSUMPTION 1.2.1:** The sole admitted analytic representations are those in the fixed reference frame of the experimenter without the use of integrating factors, called direct analytic representations.

Only after direct representations have been identified, the use of the transformation theory may have physical reference. Due to its importance, the above assumption will also be adopted throughout this monograph.

As an illustration, the admission of integrating factors within the fixed coordinate system of the experimenter does not allow the achievement of an analytic representation without external terms of a restricted class of nonconservative systems, resulting in Hamiltonians of the type \( H = e^{\epsilon_{(\text{form})}} \times \frac{p^2}{2} + \epsilon. \) This
Hamitonian has a fully valid conventional meaning of representing the time evolution. However, this Hamiltonian loses its meaning as representing the energy of the system. The quantization of such a Hamiltonian then leads to a plethora of illusions, such as the belief that the uncertainty principle for energy and time is still valid while, for the example here considered, such a belief has no sense because $H$ does not represent the energy (see Ref. [96] for more details).

Under the strict adoption of Assumption 1.2.1, all these ambiguities are about because $H$ will always represent the energy, irrespective of whether conserved or not conserved, thus setting up solid foundations for correct physical interpretations.

1.2.3 General Inapplicability of Conventional Mathematical and Physical Methods for Interior Dynamical Systems

The impossibility of reducing interior dynamical systems to an exterior form within the fixed reference frame of the observer causes the loss for interior dynamical systems of all conventional mathematical and physical methods of the 20th century.

To begin, the presence of irreducible nondiagonal external terms in the analytic equations causes the loss of their derivability from a variational principle. In turn, the lack of an action principle and related Hamilton-Jacobi equations causes the lack of any possible quantization, thus illustrating the reasons why the voluminous literature in quantum mechanics of the 20-th century carefully avoids the treatment of analytic equations with external terms.

By contrast, one of the central objectives of this monograph is to review the studies that have permitted the achievement of a reformulation of Eqs. (1.2.3) fully derivable from a variational principle in conformity with Assumption 1.2.1, thus permitting a consistent operator version of Eqs. (1.2.5) as a covering of conventional quantum formulations.

Recall that Lie algebras are at the foundations of all classical and quantum theories of the 20th century. This is due to the fact that the brackets of the time evolution as characterized by Hamilton's equations,

$$\frac{dA}{dt} = [A, H] + \sum_{k=1}^{n} h_k \frac{\partial H}{\partial p_k} = \sum_{k=1}^{n} h_k \frac{\partial H}{\partial p_k} - \sum_{k=1}^{n} \frac{\partial A_k}{\partial p_k} \frac{\partial H}{\partial p_k} = \sum_{k=1}^{n} \frac{\partial}{\partial p_k} \left( A_k \frac{\partial H}{\partial p_k} \right)$$

firstly, verify the conditions to characterize an algebra as currently understood in mathematics, that is, the brackets $[A, H]$ verify the right and left scalar and distributive laws,

$$[\alpha, A] = \alpha A - A \alpha = [A, \alpha], \quad \forall \alpha \in \mathbb{C}$$

not considering any couples of constants, as required by the above property causes the inapplicability of conventional relativities for dynamical systems with resistive forces.

The scientific influence caused by the reduction of interior dynamical systems to systems of point-like particles moving in vacuum, is indeed of historical proportion because it implied the belief of the exact applicability of special relativity and quantum mechanics for all conditions of particles existing in the universe, thus implying their applicability under conditions for which these theories were not intended for.

A central scope of this monograph is to show that the imposition of said theories to interior dynamical systems causes the suppression of new clean energies and fuels already in industrial, let alone scientific, development, thus raising serious problems of scientific ethics and accountability.

Note that the above property causes the inapplicability of conventional relativities for the description of the external constituents of interior dynamical systems, let alone their description as a whole.

1.2.4 Inapplicability of Special Relativity for Dynamical Systems with Resistive Forces

The scientific influence caused by the reduction of interior dynamical systems to systems of point-like particles moving in vacuum, is indeed of historical proportion because it implied the belief of the exact applicability of special relativity and quantum mechanics for all conditions of particles existing in the universe, thus implying their applicability under conditions for which these theories were not intended for.

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The physical nature of the argument, particularly when professed by experts, is established by numerous experimental evidence reviewed in the this section.

Figure 1.6. A further visual evidence of the lack of applicability of Einsteinian doctrines within physical media is the reduction of light in water, due to the divergence of its speed in water, thus leaving about 30% of the reduction quantitatively unexplained. Note the use of the word "inapplicable", rather than "violated" or "broken". The above evidence establishes beyond credible doubt the following:

THEOREM 1.2.2 [10b]: Galileo's and Poincaré symmetries are inapplicable for classical and operator interior dynamical systems due to the lack of Keplerian structure, the presence of contact, zero-range, non-potential interactions, and other reasons.

The nonscientific character of the above view is established by the following evidence known to experts to qualify as such:

1) Photons are neutral, thus having a high capability of penetration within electronic clouds, or, more technically, the scattering of photons on atomic electronic clouds (called Compton scattering) is rather small. Empirical calculations (that can be done by a first year graduate student in physics via quantum electrodynamics) show that, in the most optimistic of the assumptions and corrections, said scattering can account for only 3% of the reduction of the speed of light in water, thus leaving about 70% of the reduction quantitatively unexplained. Note that the deviation from physical reality is of such a magnitude that it cannot be "resolved" via the usual arbitrary parameters "to make things fit."

2) The reduction of speed occurs also for radio waves with one meter wavelength propagating within physical media, in which case the reduction to photons has no credibility due to the very large value of the wavelength compared to the size of atoms. The impossibility of a general reduction of electromagnetic waves to photons propagating within physical media is independently confirmed by the existence of vast experimental evidence on non-Doppler's effects reviewed in Chapter 9 indicating the existence of contributions outside the Doppler's law even when adjusted to the local speed.

3) There exist today a large volume of experimental evidence reviewed in Chapter 5 establishing that light propagates within hyperdense media, such as those in the interior of hadrons, nuclei and stars, at speed much bigger than the speed in vacuum,

\[ C = c \cdot \theta \cdot e^{C \cdot c} \cdot n \ll 1. \quad (1.2.13) \]

where \( \theta \) is a factor of reduction of light from the speed in water, \( C \cdot c \) is a measure of the speed of light in the medium, and \( n \) is the density.

In conclusion, experimental evidence beyond credible doubt has established that the speed of light \( C \) is a local quantity dependent on the characteristics in which the propagation occurs, with speed \( C = c \cdot \theta \) in vacuum, speed \( C \cdot c \cdot e^{C \cdot c} \cdot n \approx c \) within medium of very high density.

The variable character of the speed of light then make the lack of universal applicability of Einsteinian doctrines, once the latter are notoriously based on the philosophical assumption of "universal constancy of the speed of light".

1.2.6 Inapplicability of the Galilean and Poincaré symmetries for Interior Dynamical Systems

By remaining at the classical level, the inapplicability of Einsteinian doctrines within physical media is additionally established by the dramatic dynamical differences between the structure of a planetary system such as our Solar system, and the structure of a planet such as Jupiter.

The planetary system is a Keplerian system, that is, a system in which the heaviest component is at the center (actually in one of the two foci of elliptical orbits) and the other constituents orbit around it without collision.

The inapplicability of the Galilean and Poincaré symmetries then implies the exiting from all boundaries of credibility, let alone of science.

The above evidence establishes beyond credible doubt the following:

THEOREM 1.2.3 [10b]: Galileo's and Poincaré symmetries are inapplicable for classical and operator interior dynamical systems due to the lack of Keplerian structure, the presence of contact, zero-range, non-potential interactions, and other reasons.

Note the use of the word "inapplicable", rather than "violated" or "broken". This is due to the fact that, as clearly stated by the originators of the basic spacetime symmetries (rather than their followers of the 20th century), Galileo's and Poincaré symmetries were not built for interior dynamical conditions. Perhaps the biggest scientific imbalance of the 20th century has been the abstraction of hadronic constituents to point-like particles as a necessary condition to use conventional spacetime symmetries, relativities and quantum mechanics.

In conclusion, experimental evidence beyond credible doubt has established that light propagates within hyperdense media, such as those in the interior of hadrons, nuclei and stars, at speed much bigger than the speed in vacuum.

COROLLARY 1.2.3A [10b]: Classical Hamiltonian mechanics and related Galilean and special relativities are not exactly valid for the treatment of interior-clausal systems such as the structure of Jupiter, while nonrelativistic and relativistic quantum mechanics and related Galilean and special relativities are not exactly valid for interior particles systems, such as the structure of hadrons, nuclei and stars. Another important scope of this monograph is to show that the problem of the exact spacetime symmetries applicable to interior dynamical systems is not a mere academic issue, because it carries a direct societal relevance. In fact,
we shall show that bosonic spacetime symmetries specifically built for interior systems predict the existence of new clean energies and fuels that are prohibited by the spactetime symmetries of the exterior systems.

As we shall see in Section 1.2.7, Chapter 6 and Chapter 12, the assumption that the undetectable quarks are physical constituents of hadrons is highly probable, and so new energy based on processes occurring in the interior of hadrons (rather than in the interior of their ensembles such as nuclei), can be generated. The reason that the conjecture of hadronic constituents that can be fully defined in our spacetime and can be produced free under suitable conditions, directly implies new clean energies.

1.2.7 The Scientific Imbalance Caused by Quark Conjectures

One of the most important objectives of this monograph, culminating in the presentation of Chapter 12, is to show that the conjecture that quarks are physical particles existing in our spacetime constitutes one of the biggest threats to mankind because it prevents the orderly scientific process of resolving increasingly catastrophic environmental problems.

It should be clarified in this respect, as repeatedly stated in the author in his writings that the structure, Hadron-type, SU(3)-value classification of hadrons into families can be reasonably considered as having a final character (see e.g., Ref. [99] and papers quoted therein), in view of the historical capability of said classification to predict several new particles whose existence was subsequently verified experimentally. All doubts herein considered solely refer to the joint use of the same classification models as providing the structure of each individual element of a given hadronic family (for more details, see memoirs [100,101] and present [102] and Chapter 6).

Far from being alone, this author has repeatedly expressed the view that quarks cannot be physical constituents of hadrons existing in our spacetime for numerous independent reasons.

Historically, the study of nuclei, atoms and molecules required two different models: one for the classification and a separate one for the structure of the individual elements of a given SU(3)-value family. Quark theories depart from this historical teaching because of their conception to represent in a single theory both the classification and the structure of hadrons.

As an example, the idea that the Mendeleev classification of atoms could jointly provide the structure of each individual atom of a given valence family is outside the boundary of science. The Mendeleev classification was essentially achieved via classical theories, while the understanding of the atomic structure required the construction of a new theory, quantum mechanics.

Independently from the above dichotomy classification vs structure, it is well known by specialists, but rarely admitted, that quarks are purely mathematical constructs.

energy equivalence $E = mc^2$ for any particle composed of quarks, against vast experimental evidence to the contrary.

4) Even assuming that, because some threat of scientific manipulation, the above inclusions are received, it is known by experts that quark theories have failed to achieve a representation of all characteristics of protons and neutrons, with catastrophic inconsistencies in the representation of spin, magnetic moment, mass, charge, radii and other basic features [102].

5) It is also known by experts that the application of quark conjectures to the structure of nuclei has multiplied the controversies in nuclear physics, resulting none of them. As an example, the assumption that quarks are the constituents of the protons and the neutrons constituting nuclei has failed to achieve a representation of the main characteristics of the simplest possible nucleons, the deuterons. In fact, quark conjectures are afflicted by the catastrophic inconsistencies of being unable to represent the spin 1 of the deuteron (since they predict spin zero in the ground state while the deuteron has spin 1), unable to represent the anomalous magnetic moment of the deuteron, unattainable to represent the charge radius of the deuteron, and when passing to larger nuclei, such as the strontium, the catastrophic inconsistencies of quark conjectures can only be defined as being embarrassing [102].

In summary, while the final character of the SU(3)-value classification of hadrons into families has reached a value beyond scientific doubt, the conjecture that quarks are physical particles existing in our spacetime constitutes one of the biggest threats to mankind because it prevents the orderly scientific process of resolving increasingly catastrophic environmental problems.

The decay of scientific ethics in the field is so serious, and the implications for mankind so potentially catastrophic (due to the suppression by quark conjectures as physical particles of possible new clean energies studied in Volume II) that, in the author’s view, quark conjectures have been interspersed to scientific ethics, and so far as the scientific community in favor of neutrino conjectures conceived and implemented in a capillary and technical way to the detriment of all mankind because it prevents the orderly scientific process of resolving increasingly catastrophic environmental problems.

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As we shall see in Section 1.2.7, Chapter 6 and Chapter 12, the assumption that quarks are physical particles is afflicted by a plethora of major problematic aspects today known to experts as catastrophic inconsistencies of quark conjectures, such as:

1) No particle possessing the peculiar features of quark conjectures (fraction change, etc.), has ever been detected to date in any high energy physical laboratory around the world. Consequently, a main consistency requirement of quark conjectures is that quarks cannot be produced free and, consequently, they must be "permanently contained" in the interior of hadrons. However, it is well known to experts that, despite half a century of attempts, not truly convincing "quark confinement" inside protons and neutrons has been achieved; one can be expected on serious scientific grounds by assuming (as it is the case of quark conjectures) that quark mechanics is identical inside and outside hadrons. This is due to a plethora of quantum mechanics, Heisenberg’s uncertainty principle, according to which, given any manipulated theory appearing to show confinement for a given quark, a graduate student in physics can always prove the construction of a new theory, quark mechanics.

It should be stressed in this respect, as repeatedly stated by the author in his writings, that neutrinos cannot be detected. Hence, the scientifically correct statement would be the “detection of physical particles predicted by neutrino conjectures.” As it was the case for Murray Gell-Mann, it is unfortunate that such a serious scientific position of the same classification models as providing the structure of each individual element of a given hadronic family is outside the boundary of science. The Mendeleev classification was essentially achieved via classical theories, while the understanding of the atomic structure required the construction of a new theory, quantum mechanics.

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in Italy to detect neutrinos coming from the opposite side of Earth (since the same classification being admitted by theories that require no quarks at all as indicated theories. This occurrence is, per se, controversial. For instance, contro-
discovered one or another quark, while in reality the laboratories discovered
voiced by large collaborations.
physical particles, as we shall indicate in Chapter 6.

In the 1980s, a large laboratory was built deep into the Gran Sasso mountain
with implications sufficient to require studies over the entire third millennium.
The existence of the neutron was subsequently confirmed experimentally in 1932 by Chadwick [105]. However, numerous objections were raised by the leading
physicists of the time against Rutherford’s conception of the neutron as a bound
state of one proton $p$ and one electron $e$.
Pauli [106] first noted that Rutherford’s synthesis violates the angular momen-
tum conservation law because, according to quantum mechanics, a bound state
of two particles with spin 1/2 (the proton and the electron) must yield a particle
with integer spin and cannot yield a particle with spin 1/2 and charge zero such
as the neutron. Consequently, Pauli conjectured the existence of a new neutral
particle in our spacetime. Consequently, the exclusion of the so-called 
little neutron” or 

$$n: p + e^+ → n + e^-,$$

By the middle of the 20th century there was an dear experimental evidence
acceptable by the scientific community at large confirming the neutrino-conjecture
beyond doubt, except for experimental claims in 1959 that are known today to be
basically flawed as various ground were we shall see below and in Chapter 6.
In the last part of the 20th century, there was the advent of the so-called

$$\text{SU}(3)$$
described and related quark conjectures studied in the preceding
subsection. In this way, neutrino conjectures became deeply linked to and their
prediction intrinsically based on quark conjectures.

This event provided the first fatal blow to the credibility of the neutrino con-
jectures because serious physics cannot be done via the use of conjectures based
on other conjectures.

In fact, the marriage of neutrino and quark conjectures within the standard
model has requested the multiplication of neutrinos, from the neutrino and anti-
neutrino conjectures of the early studies, to six different hypothetical particles,
the so called, lepton, meson and tau neutrinos and their antiparticles. In the
absence of these particles the standard model would maintain its meaning as
classification of hadrons, but would lose in an irrevocable way the joint capa-
ibility of providing also the structure of each particle in a hadronic multiplet.

In turn, the multiplication of the neutrino conjectures has requested the ad-
ditional conjecture that the electron, muon and tau neutrinos have masses and,
since the latter conjecture resulted in being insufficient, these was the need for the
additional conjectures that neutrinos classify different masses, as necessary to
salvage the structural features of the standard model. Still in turn, the lack of
resolution of the prevailing conjectures has requested the definite conjectures
that neutrinos oscillate, namely, “that they change flavor” (transform among
themselves back and forth).

These more recent experiments resulted in claims that, on strict scientific
grounds, should be considered “experimental beliefs” by any serious scholars for
numerous reasons, such as:
1) The predictions are based on a litany of sequential conjectures none of which
is experimentally established on clear ground;
2) The theory contains a plethora of unrestricted parameters that can essentially
fit any pre-set data (see next subsection);
3) The “experimental results” are based on extremely few events cut out of hun-
dreds of millions of events over years of tests, thus being basically insufficient
in number for any serious scientific claim;
4) In various cases the “neutrino detector” include radioactive isotopes that
can themselves account for the selected events;
5) The interpretation of the experimental data via neutrino and quark conjec-
tures is not unique, since there exist nowadays other theories representing exactly
the same events without neutrinos and quark conjectures (including a basically
new scattering theory of nonlocal type indicated in Chapter 3 and, more exten-
sively, in monographs [103]).

To understand the scientific scene, the serious scholar (that is, the scholar who
politically aligned to the preferred “pet theories” indicated in the Preface) should
note that neutrinos and quark conjectures have requested to date the expenditure
of over one billion dollar of public and private funds, thus leading to the theoretical
and experimental research with the result of increasing the controversies rather
than resolving any of them.

Furthermore, it is now time for a new form of reflection. Scientific ethics and
accountability require that serious scholars in the field exercise caution prior to
venturing claims of actual physical existence of so controversial and directly
unverifiable conjectures.

Such a moment of reflection requires the re-inspection of the neutrino conjec-
tures at its foundation. In fact, it is important to dispose the neutrino conjecture
as originally conceived, and then dispose the flawed extension of the conjecture
as requested by quark conjectures.

As reported in nuclear physics textbooks (see, e.g., Ref. [13]), the energy
experimentally measured as being carried by the electron in beta decays is a
bell-shaped curve with a maximum value of 0.802 MeV, that is the difference in
value between the mass of the neutron and that of the resulting proton in the

Figure 1.8 A schematic illustration of the fact that the electron in beta decays can be emitted
different directions. When the energy in the beta decay is computed with the inclusion of
the Coulomb interactions between the expelled (negatively charged) electron and the (positively
charged) nucleus at different emission directions, the nucleus acquires the “missing energy,”
without any energy left for the hypothetical neutrino. As we shall see in Chapter 6, rather than
being a disaster, the occurrence is at the foundation of a possible basically new scientific
fortun with implications sufficient to require studies over the entire third millennium.

troversies have occurred in regard to experimental claims of neutrino detection
voiced by large collaborations.

To begin, both neutrinos and quarks cannot be directly detected as physical
particles in our spacetime. Consequently, all claims on their existence are indi-
cert, that is, based on the detection of actual physical particles predicted by the
indicated theories. This occurrence is, per se, controversial. For instance, contro-
versies are still raging following announcements by various laboratories to have
“discovered” one or another quark, while in reality the laboratories discovered
physical particles predicted by a Mendeleev-type classification of particles, the
same classifications being admitted by theories that require no quarks as all
physical particles, as we shall indicate in Chapter 6.

In the 1960s, a large laboratory was built deep into the Gran Sasso mountain
in Italy to detect neutrinos coming from the opposite side of Earth (since the
mountain was used as a shield against cosmic rays). Following the investment
of large public funds and five years of tests, the Gran Sasso Laboratory released no
evidence of clear detection of neutrino originated events.

Hence, in order to pass to a scientific conclusion in the use of public funds, the fail-
sure of the Gran Sasso experiments to produce any neutrino evidence-stimulated
should be written ($p^-, e^- → n + s$, where $s$ is the neutrino, in which case the inverse reaction
the spontaneous decay of the neutron) reads $n → p^+ + e^- + s$, where $s$ is the antineutrino.

Despite the scientific authority of historical figures such as Pauli and Fermi, the
conjecture on the existence of the neutrino and antineutrino as physical particles
was never universally accepted by the entire scientific community because of: the
impossibility for the neutrinos to be directly detected in laboratory; the neutrinos’
ability to interact with matter in any appreciable way; and the existence of
alternative theories that do not need the neutrino conjecture (see Refs. [109-110] and
literature quoted therein, plus the alternative theory presented in Chapter 6).
Moreover, such an attraction is clearly dependent on the angle of emission of the electron from the electron-neutrino paring electron.

The maximal value of the energy occurs for radial emissions of the electron, the minimal value occurs for tangential emissions, and the intermediate value occurs for intermediate directions of emissions, resulting in the experimentally detected bell-shaped curve of Figure 1.7.

When the calculations are done without political alignments or pre-existing doctrines, it is easy to see that the "missing energy" in beta decays is entirely absorbed by the neutrino via its Compton interaction with the emitted electron. Consequently, in beta decay there is no energy at all available for the neutrino conjecture, by reaching in this way a disposal of the conjecture itself at the historical origination.

Supporters of the neutrino conjecture are expected to present as counter-arguments various counter-arguments on the lack of experimental evidence for the neutrino to acquire said "missing energy." Before doing so, said supporters are suggested to exercise scientific caution and study the new structure models of the neutrino without the neutrino conjecture (Chapter 6), as well as the resulting new structure models of nuclei (Chapter 7) and the resulting new clean energies (Chapter 12). Only then, depending on the strength of their political alignment, they may eventually realize that, in abusing academic authority to perpetuate unproved neutrino conjectures they may eventually be part of real crimes against mankind.

The predictable conclusion of this study is that theoretical and experimental research on neutrinos and quark conjectures should indeed continue. However, theoretical and experimental research on theories without neutrino and quark conjectures and their new clean energies should be equally supported to prevent a clear espionage of scientific democracy on fundamental need of mankind, evident problems of scientific accountability, and a potentially severe judgment by posterity.

For technical details on the damage caused to mankind by the current lack of serious scientific caution on neutrino conjectures, interested readers should study Volume II and inspect the Open Denunciation of the Nobel Foundation for Bestow an Organized Scientific Obscurantism available in the web site http://www.scientificaethics.org/Nobel-Foundation.htm.

1.2.9 The Scientific Imbalance in Experimental Particle Physics

Another central objective of this monograph is to illustrate the existence at the dawn of the third millennium of a scientific obscuration of unprecedented proportions, caused by the manipulation of experimental data on the use of experimentally unproved and actually unverifiable quark conjectures, neutrino conjectures and other conjectures complemented by a variety of ad hoc parameters for the unspoken, but transparent reason that, in the absence of the basic axioms of relativistic quantum mechanics and special relativity in particle physics.

As we shall see in technical details in Chapter 5, the quantum axiom of experimental data is that, in order to safeguard the theory and its underlying Einsteinian doctrines, organized interests introduced "lost" of key parameters deprived of any physical meaning, or origin, and then claim the exact validity of ad hoc decisions. The scientific truth is that the four ad hoc parameters are merely a direct manifestation of the deviation from the basic axioms of relativistic quantum mechanics and special relativity in particle physics.

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The scientific truth is that the four ad hoc parameters are merely a direct manifestation of the deviation from the basic axioms of relativistic quantum mechanics and special relativity in particle physics.

Figure 1.10: A schematic view of the Bose-Einstein correlation originating in proton-antiproton annihilations, for which the predictions of relativistic quantum mechanics are dramatically far from experimental data from unadulterated first principles. In order to safeguard the theory and its underlying Einsteinian doctrines, organized interests introduced "lost" of key parameters deprived of any physical meaning, or origin, and then claim the exact validity of ad hoc decisions.

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The scientific truth is that the four ad hoc parameters are merely a direct manifestation of the deviation from the basic axioms of relativistic quantum mechanics and special relativity in particle physics.
experts, to qualify as such, know that the representation requires a structural modification of the basic axiom of expectation values as well as for numerous additional reasons, such as:

1. The Bose-Einstein correlation is necessarily due to contact, nonpotential, nonlocal-integral effects originating in the deep overlapping of the hyperdense charge distributions of protons and antiprotons inside the fireball.

2. The mathematical foundation of quantum mechanics (such as its topology), let alone its physical laws, are incapable of a meaningful representation of said nonlocal and nonpotential interaction as outlined in preceding sections; and

3. Special relativity is also inapplicable, e.g., because of the inapplicability of the basic Lorentz and Poincaré symmetries due to lack of a Keplerian structure, the approximate validity of said theories remaining beyond scientific doubt.

Admittedly, there exist a number of semiphenomenological models in the literature capable of a good agreement with the experimental data. Scientific deceptions occur when these models are used to claim the exact validity of quantum mechanics and special relativity since the representations of experimental data require necessary structural departures from basic quantum axioms.

Of course, the selection of the appropriate generalization of quantum mechanics and special relativity for an exact representation of the Bose-Einstein correlation is open to scientific debate. Scientific deceptions occur when the need for such a generalization is denied for personal gains.

As we shall see, relativistic hadronic mechanics provides an exact and invar- iant representation of the experimental data of the Bose-Einstein correlation at high and low energies via unaltered basic axioms, by providing in particu-
lar a direct representation of the shape of the $p - \bar{p}$ fireball and its density, while recovering the basic invariant under a broader realization of the Poincaré symmetry.

An in-depth investigation of all applications of quantum mechanics and special relativity at large events that they have provided an exact and invariant represen-
tation from unaltered basic axioms of all experimental data of the hyperdense atom, as well as of physical conditions in which the mutual distance of particles is much larger than the size of the charge distribution (for hadrons) or of the wavepackets of particles (for the case of the electron).

1.2.10 The Scientific Imbalance in Nuclear Physics

There is no doubt that quantum mechanics and special relativity permitted histor- ical advances in also nuclear physics during the 20th century, as illustrated, for instance, by nuclear power plants. However, any claim that quantum mechanics and special relativity are exactly valid in nuclear physics is a scientific deception, particularly when professed by experts, because of the well-known inability of these theories to achieve an exact and invariant representation of numerous nu-

The first evidence on the lack of exact character of quantum mechanics in nuclear physics dates back to the birth of nuclear physics in the 1930s where it emerged that experimental values of nuclear magnetic moments could not be explained with quantum mechanics, because, starting with small deviations for small nuclei, the deviations then increased with mass, to reach deviations for large nuclei, such as the Zr as large as to escape any use of unknown parameters “to fix things” (see Figure 1.8).

Subsequently, it became clear that quantum mechanics and special relativity could not explain the simplest possible nucleus, the deuteron, despite vast efforts. In fact, quantum mechanics missed about 1% of the deuteron magnetic moment explained by quantum mechanics and special relativity to this day. The time has long come to stop adding potentials and semi-phenomenological fits were reached, and quantum mechanics and special relativity were again claimed to be exact in nuclear physics, while in the scientific reality the used parameters are a direct representation of deviations from the basic axioms of the theories as shown in detail in Chapter 5.

Subsequently, when the use of arbitrary parameters failed to achieve credible representations of nuclear data (such as nuclear magnetic moments as indicated below), organized academic interests claimed that “the deviations are resolved by deeper theories such as quark theories.” As that point nuclear physics left the qualification of a true science to become a scientific religion.

Besides a plethora of intimate problematic aspects or clear inconsistencies (such as the impossibility for quarks to have gravity mentioned earlier), quark

...
As we shall see in Chapter 7, a central objective of hadronic mechanics is that of truncating the addition of potentials and re-summing instead the nuclear force from its analytic formulations, by first separating potential nonpotentiel forces, and then examining in details each of them.

In summary, the lack of exact character of quantum mechanics and special relativity in nuclear physics is beyond scientific doubt. The open scientific issue is the selection of the appropriate generalization, but not its need.

As we shall see in Chapter 6, the covering hadronic mechanics and nonpotional relativity resolve the fundamental open problems of nuclear physics by permitting the industrial development of new clean energies based on light natural and stable elements without the emission of dangerous radiations and without the release of radioactive wastes.

1.2.11 The Scientific Imbalance in Superconductivity

The condition of superconductivity in the 20th century can be compared to that of atomic physics prior to the representation of the structure of the atom.

Recall that individual electrons cannot achieve a superconducting state because their magnetic fields interact with electromagnetic fields of atoms by creating in this way what we call electric resistance. Superconductivity is instead reached by deeply correlated-bound pairs of electrons in singlet couplings, called Cooper pairs. In fact, these pairs have an essentially null total magnetic field (due to the opposite orientations of the two fields), resulting in a substantial decrease of electric resistance.

There is no doubt that quantum mechanics and special relativity have permitted the achievement of a good description of an "ensemble" of Cooper pairs, although each Cooper pair is necessarily abstracted as a point, the latter condition being necessary from the very structure of the theories.

However, it is equally well known that quantum mechanics and special relativity have been unable to reach a final understanding and representation of the structure of one Cooper pair, trivially, because electrons repel each other according to the fundamental Coulomb law.

The failure of basic axioms of quantum mechanics and special relativity to represent the attractive force between the two identical electrons of the Cooper pairs motivated the hypothesis that the attraction is caused by the exchange of a new particle called phonon. However, phonons certainly exist in sounds, but they have found no verification at all in particle physics, thus remaining purely mathematical.

In reality, as we shall see in Chapter 7, the interactions underlying the Cooper pairs are of purely contact, nonlocal and integral character due to the mutual penetration of the wavepackets of the electrons, as depicted in Figure 1.10. As such, they are very similar to the interactions responsible for Pauli's exclusion principle in atomic structures.

Under these conditions, the granting of a potential energy, as necessary to have phonons exchanges, is against physical evidence, as confirmed by the fact that any representation of Pauli's exclusion principle via potential interactions causes sizable deviations from spectral lines.

Therefore, the belief that quantum mechanics and special relativity provide a complete description of superconductivity is pure academic politics deprived of scientific content.

Superconductivity is yet another field in which the exact validity of quantum mechanics and special relativity has been stretched in the 20th century well beyond its limit for known political reasons. At any rate, superconductivity has exhausted all its productive capacities, while all advances are attempted via empirical trials and errors without a guiding theory.

As it is the case for particle and nuclear physics, the lack of exact character of quantum mechanics and special relativity in superconductivity is beyond doubt. Equally beyond doubt is the need for a deeper theory.

As we shall see in Chapter 7, the covering hadronic mechanics and special relativity provide a quantitative representation of the structure of the Cooper pair in excellent agreement with experimental data, and with basically novel predictive capabilities, such as the industrial development of a new electric current, that is characterized by correlated electron pairs in single coupling, rather than electrons.

1.2.12 The Scientific Imbalance in Chemistry

There is no doubt that quantum chemistry permitted the achievement of historical discoveries in the 20th century. However, there is equally no doubt that the widespread assumption of the exact validity of quantum chemistry caused a large scientific imbalance with vast implications, particularly for the alarming environmental problems.

After about one century of attempts, quantum chemistry still misses a historical 2% of molecular binding energies when derived from axiomatic principles without ad hoc substitutions (see below). Also, the deviations for electric and magnetic moments are embarrassing not only for their numerical values, but also because they are wrong even in their sign [14], not to mention numerous other inconsistencies outlined below.

It is easy to see that the reason preventing quantum chemistry from being exactly valid for molecular structures is given by contact, nonlocal-integral and nonpotential interactions due to deep wave-overlapping in valence bonds that, as such, are beyond any realistic treatment by local-differential-potential axioms, such as those of quantum chemistry (Figure 1.10).

Figure 1.11. A first clear evidence of the lack of exact validity of quantum chemistry. The top view depicts one hydrogen atom for which quantum mechanics resulted in being exactly valid. The bottom view depicts two hydrogen atoms coupled into the H2 molecule in which two quantum chemistry has historically missed a 2% of the binding energy when applied without ad hoc axioms "to fit things" (such as the need of the screening of the Coulomb and then claim that quantum chemistry is exact). Since mixing does not participate in the molecular bond, the origin of the insufficiency of quantum mechanics and chemistry rests in the valence bond.

By no means the above insufficiencies are the only ones. Quantum chemistry is afflicated by a true litany of limitations, insufficiencies or sheer inconsistencies that constitute the chief secret of the demimay of the 20th century become known to experts (since they have been published in refereed journals), but they remain generally ignored evidently for personal reasons.

We outline below the insufficiencies of quantum chemistry for the simplest possible class of systems, those that are isolated from the rest of the universe, thus verifying conventional conservation laws of the total energy, total linear momentum, etc., and are reversible (namely, their time reversal image is as physical as the original system).

The most representative systems of the above class are given by molecules, here generically defined as aggregates of atoms under a valence bond. Despite undeniable achievements, quantum chemical models of molecular structures have the following fundamental insufficiencies studied in detail in monograph [11]:

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1: Quantum chemistry lacks a sufficiently strong molecular binding force. After 150 years of research, chemistry has failed to identify to this day the attractive force needed for a credible representation of valence bonds. In the absence of such an attractive force, names such as “valence” are pure nomenclatures without quantitative meaning.

To begin, the average of all Coulomb forces among the atoms constituting a molecule is identically null. As an example, the currently used Schrödinger equation for the H$_2$ molecule is given by the familiar expression [15],

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + \frac{e^2}{r_{12}} + \frac{e^2}{r_{13}} + \frac{e^2}{r_{23}} \right) \psi = E \psi,$$  

(1.2.20)

which equation contains the Coulomb attraction of each electron by its own nucleus, the Coulomb repulsion of the two electrons, and the Coulomb repulsion of the two protons.

It is easy to see that, in semiclasical average, the two attractive forces of each electron from the nucleus of the other atom, the Coulomb repulsion of the two electrons, and the Coulomb repulsion of the two protons.

Irrespective from the above, a first year graduate student in chemistry can precisely to introduce the missing force, today known as the strong nuclear force, that is, firstly, ATTRACTIVE, secondly, sufficiently STRONG, and, thirdly, INVARIANT. The exact and invariant representation of molecular data will then be a mere consequence.

2: Quantum chemistry admits an arbitrary number of atoms in the hydrogen, water and other molecules. This inconsistency is proved beyond scientific doubt by the fact that the exchange, van der Waals, and other forces used in current molecular models were conceived in nuclear physics for the primary purpose of admitting a large number of constituents. When the same forces are used for molecular structures, they also admit an arbitrary number of constituents. As specific examples, when applied to the structures of the hydrogen or water molecule, any graduate student in chemistry can prove that, under exchange, van der Waals and other forces of nuclear type, the hydrogen, water and other molecules admit an arbitrary number of hydrogen atoms (see Figure 1.13).

Rather than explaining the reason why nature has selected the molecules H$_2$, H$_2$O, etc. as the sole possible molecular structures, and other structures such as H$_2$, H$_2$O, HO, HO, ROH, ROH, etc. cannot exist. As we shall see in Chapter 8, the “strong nuclear force” permitted by hadronic chemistry can rule out among “pairs” of valence electrons, thus resolving the historical problem in a quantitative way.
screened Coulomb laws of type (1.2.22). However, such screenings are solely admitted in the molecular-integral region of the water molecule and involve unbounded valence electrons that are of the order of 10^{-11} cm, while recovering the conventional Coulomb law automatically for all distances greater than 1Å. This concept permits the achievement of an exact representation of molecular binding energies while preserving in full the quantum structure of the individual atoms.

5: Quantum chemistry cannot provide a meaningful representation of thermonuclear reactions. The missing 2% in the representation of binding energies is misleadingly small, because it corresponds to about 1,000 Kcal/mole while an ordinary thermonuclear reaction (such as that of the water molecule) implies an average of 40 Kcal/mole. No scientific calculation can be conducted when the error is of about twenty times the quantity to be computed.\footnote{As we shall see in Chapter 8, our covering hadronic chemistry does indeed permit exact thermochimical calculations because it has achieved exact representations of molecular characteristics.}

As we shall see also in Chapter 8, our covering hadronic chemistry will also resolve this additional insufficiency because the mechanism permitting the exact representation of molecular characteristics implies a fast convergent lifting of conventional slowly convergent series.

6: Computer usage in quantum chemical calculations requires excessively long periods of time. This additional, well known insufficiency is not only due to the slow convergence of conventional quantum series, an insufficiency that persists to this day despite the availability of powerful computers. As we shall also see in Chapter 8, our covering hadronic chemistry will also resolve this additional insufficiency because the mechanisms permitting the exact representation of molecular characteristics implies a fast convergent lifting of conventional slowly convergent series.

7: Quantum chemistry predicts that all molecules are paramagnetic. This inconsistency is a consequence of the most rigorous discipline of the 20th century, quantum electrodynamics, establishing that, under an external magnetic field, the orbits of peripheral atomic electrons must be oriented in such a way as to offer a magnetic polarity opposite to that of the external field (a polarization that generally occurs via the transition from a three-dimensional to a toroidal distribution of the orbitals).

According to quantum chemistry, atoms belonging to a molecule preserve their individuality. Consequently, quantum electrodynamics predicts that the peripheral atomic electrons of a molecule must acquire polarized orbitals under an external magnetic field.

As a result, quantum chemistry predicts that the application of an external magnetic field to hydrogen $H_2$, to oxygen $O_2$, to water $H_2O$, and to other molecules implies their acquisition of a net total, opposite polarity, $H_1 - H_2$, $H_1 - O_1$, $H_1 - O_2$, etc., which polarization is in dynamic disregard with respect to the external field.

The above inconsistency can also be derived from its inability to restrict the correlation solely to valence pairs. By contrast, the strong valence bond of the covering hadronic chemistry eliminates the independence of individual atoms in a molecular structure, by correctly representing the diamagnetic or paramagnetic character of substances.

No serious advances in chemistry can occur without, first, the admission of the above serious inconsistencies and/or insufficiencies, secondly, their detailed study, and, thirdly, their resolution via a covering theory.
In summary, under the above deviations, any use of quantum mechanics, superconductivity and chemistry for the study of enhanced electric arcs etc., the boundaries of scientific ethics and accountability. The departure of experimental evidence from old doctrines are just too big to be removed via arbitrary parameters “to fix things”, thus mandating the construction of suitable covering theories.

1.3 THE SCIENTIFIC IMBALANCE CAUSED BY NON-REVERSIBILITY

1.3.1 The Scientific Imbalance in the Description of Natural Processes

Numerous basic events in nature, including particle decays, such as
\[ n \rightarrow p^+ + e^- + \bar{\nu} \]  
(1.3.1)
nuclear transmutations, such as
\[ C(6,12) + \bar{H}(1,2) \rightarrow N(7,14) \]  
(1.3.2)
chemical reactions, such as
\[ H_2 + \frac{1}{2} O_2 \rightarrow H_2O \]  
(1.3.3)
and other processes are called irreversable when their images under time reversal, \( t \rightarrow -t \), are prohibited by causality and other laws. Systems are instead called reversible when their time reversal images are as casual as the original ones, as it is the case for planetary and atomic structures when considered isolated from the rest of the universe.

Yet another large scientific imbalance of the 20-th century has been the treatment of irreversible systems via the formulations developed for reversible systems, such as Lagrangian and Hamiltonian mechanics, quantum mechanics and chemistry and special relativity. In fact, all these formulations are strictly reversible, in the sense that all their basic axioms are fully reversible in time, by causing in this way limitations in virtually all branches of science.

The imbalance was compounded by use of the truncated Lagrange and Hamilton equations (see Section 1.2.2) based on conventional Lagrangians or Hamiltonians,
\[ L = \sum_{a=1,2} \left( \frac{1}{2} m_a \dot{r}_a^2 - V(r_a) \right); \]
(1.2.4a)
\[ H = \sum_{a=1,2} \frac{p_a^2}{2m_a} + V(r_a); \]  
(1.2.4b)
and therefore, all known Hamiltonians, and are applicable at all levels of study, from Newtonian mechanics to second quantization. The property established by the above theorems dismisses all nonscientific beliefs on irreversibility, and identify the real needs, the construction of formulations that are strictly irreversible, that is, irreversible for all known reversible potentials, Lagrangians or Hamiltonians, and are applicable at all levels of study, from Newtonian mechanics to second quantization.

The historical origin of the above imbalance can be outlined as follows. One of the most important teaching in the history of science is that by Lagrange [2], Hamilton [3], and Jacobi [4] who pointed out that irreversibility originates from contact nonsolitonic interactions not representable by a potential, for which reason they formulated their equations with external terms, as in Eq. (1.3.3).

In the planetary and atomic structures, there is no need for external terms, since all acting forces are of potential type. In fact, these systems admit an excellent approximation as being made-up of massive point masses moving in vacuum without collisions (external dynamical problems). In these cases, the historical analytic equations were “truncated” with the removal of the external terms.

In the course of the planetary and atomic models, the main scientific development of the 20-th century was restricted to the “truncated analytic equations”, without any visible awareness that they are not the equations conceived by the founders of analytic mechanics.

Therefore, the origin of the scientific imbalance on irreversibility is the general dismissal by scientists of the 20-th century of the historical teaching by Lagrange, Hamilton and Jacobi, as well as academic interests on the truncated analytic equations, such as quantum mechanics and special relativity. In fact, as outlined earlier, the use of external terms in the basic analytic equations cause the inapplicability of the mathematics underlying said theories.

It then follows that no serious scientific advance on irreversible processes can be achieved without first identifying a structurally irreversible mathematics and then the compatible generalizations of conventional theories, a task studied in details in Chapter 4.

As we shall see, contrary to popular beliefs, the origin of irreversibility results in being at the ultimate level of nature, that of elementary particles in interior conditions. Irreversibility then propagates all the way to the macroscopic level as to avoid the inconsistency of Theorem 1.3.1.

1.3.2 The Scientific Imbalance in Astrophysics and Cosmology

Astrophysics and cosmology are new branches of science that saw their birth in the 20-th century with a rapid expansion and majestic achievements. Yet, these new fields soon fell prey to organized interests in established doctrines with particular reference to quantum mechanics, special relativity and gravitation, resulting in yet another scientific imbalance of large proportions.

To begin, all interior planetary or astrophysical problems are irreversible, as shown by the very existence of entropy, and known thermodynamical laws strictly ignored by supporters of Einsteinian doctrines. This feature, alone, is sufficient to cause a scientific imbalance of large proportions because, as stressed above, irreversible systems cannot be credibly treated with reversible theories.

Also, quantum mechanics has been shown in the preceding sections to be applicable to all interior astrophysical and gravitational problems for reasons other than irreversibility. Any reader with an independent mind can then see the limitations of astrophysical studies for the interior of stars, galaxies and quasars based on a theory that is intrinsically incompatible for the problems considered.

The imposition of special relativity as a condition for virtually all relativistic astrophysical studies of the 20-th century caused an additional scientific imbalance. To illustrate its dimension and implications, it is sufficient to note that all calculations of astrophysical energies have been based on the relativistic mass-energy equivalence
\[ E = mc^2 \]  
(3.5)
namely, on the philosophical belief that the speed of light is the same for all conditions existing in the universe (this is the well-known “universal constancy of the speed of light”).

As indicated earlier, this belief has been disproved by clear experimental evidence, particularly for the case of interior astrophysical media in which the maximal causal speed has resulted to be \( C \approx c/e > c \), e, c << 1, in which case the correct calculation of astrophysical energies is given by the equivalence principle of the inescapable relativity (see Chapter 3)
\[ E = mc^2 + mc^4 \approx mc^2 \]  
(3.6)
thus invalidating current view on the “missing mass”, and others.
A further large-science imbalance in astrophysics and cosmology was caused by the imposition of general relativity, namely, by one of the most controversial theories of the 20th century became affected by problematic aspects and sheer inconsistencies as serious called catastrophic; as outlined in the next section. It is hoped these preliminary comments are sufficient to illustrate the weakness of the scientific foundations of astrophysical studies of the 20th century.

1.3.3 The Scientific Imbalance in Biology

By far one of the biggest scientific imbalances of the 20th century occurred in biology because biological structures were treated via quantum mechanics in full awareness that the systems described by that discipline are dramatically different than biological structures.

To begin, quantum mechanics and chemistry are strictly reversible, while all biological structures and events are structurally irreversible, since biological structures such as a cell or a complete organism, admit a birth, then growth and then die.

Moreover, quantum mechanics and chemistry can only represent perfectly rigid systems, as well known from the fundamental rotational symmetry that can only describe "rigid bodies". As a consequence, the representation of biological systems via quantum mechanics and chemistry implies that our body should be perfectly rigid, without any possibility of introducing deformable-elastic structures, because the latter would cause catastrophic inconsistencies with the basic axioms.

Moreover, another pillar of quantum mechanics and chemistry is the verification of total conservation laws, for which Heisenberg's equation of motion became established. In fact, the quantum time evolution of an arbitrary quantity $A$ is given by

$$i\hbar \frac{d}{dt} [A, H] = H_A - H + H_A$$

under which expression we have the conservation law of the energy and other quantities, e.g.,

$$i\hbar \frac{d}{dt} [H, H] = H - H + H = 0$$

A basic need for a scientific representation of biological structures is instead the representation of the time-rate-of-variation of biological characteristics, such as size, weight, density, etc. This identifies another structural incompatibility between quantum mechanics and biological systems.

When passing to deeper studies, the insufficiencies of quantum mechanics and chemistry emerge even more forcibly. For example, quantum theories can well represent the scope of sea shells, but not their growth in time. In fact, computer simulations [10] have shown that, when the geometric axioms of quantum mechanics and chemistry (those of the Euclidean geometry)

1.4. THE SCIENTIFIC IMBALANCE CAUSED BY GENERAL RELATIVITY AND QUANTUM GRAVITY

1.4.1 Consistency and Limitations of Special Relativity

As it is well known, thanks to historical contributions by Lorentz, Poincaré, Einstein, Minkowski, Weyl and others, special relativity achieved a majestic axiomatic consistency.

After one century of studies, we can safely identify the origins of this consistency in the following crucial properties:

1) Special relativity is formulated in the Minkowski spacetime over the field of real numbers;

2) All laws of special relativity are invariant (rather than covariant) under the fundamental Poincaré symmetry;

3) The Poincaré transformations and, consequently, all times evolutions of special relativity, are conserved at the classical level and unitary at the operator level with implications crucial for physical consistency.

Consequently, since canonical or unitary transforms conserve the unit by their very definition, special relativity admits basic units and numerical predictions that are conserved in time. After all, the quantities characterizing the dynamical equations are the Casimir invariants of the Poincaré symmetry.

As a result of the above features, special relativity has been and can be confidently applied to experimental measurements because the units selected by the experimenter do not change in time, and the numerical predictions of the theory can be tested at any desired time under the same conditions without fear of internal axiomatic inconsistencies.

It is well established at this writing that special relativity is indeed "compatible with experimental evidence" for the arena of its original conception, the classical and operator treatment of "point-like" particles and electromagnetic waves moving in vacuum. Despite historical results, it should be stressed that, as is the fate for all theories, special relativity has numerous well defined limits of applicability, whose identification is crucial for any serious study on gravitation, since

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It should be emphasized that the same "Einstein's special relativity" is political, since a scientifically correct name should be "Lorentz-Poincare-Einstein relativity." Also, it is inappropriate to refer (as not reviewed in numerous books by important writers) that Einstein ended up changing his views with Hilbert. Hilbert became the key instrument in the elimination of special relativity of 1905 and, for that reason, she has been originally listed as a co-author of that article, to acknowledge that she subsequently created the essay as appeared in print. In fact, Einstein assured he, Hilbert wrote the essay for that article to Hilbert. Similarly, it should be emphasized that Einstein ended up deleting Einstein in the 1905 article following the constant, and in the documented knowledge that Einstein's special relativity only consists of special relativity (see, e.g., the historical account by Leopold [62] or the instructive books [57,96]).

...
to state that general relativity (see, e.g., monograph [27]) has been the most controversial theory of the 20th century for a plethora of inconsistencies that have grown in time, rather than being addressed and resolved.

We now address some of the inconsistencies published by numerous scholars in refereed technical journals, yet generally ignored by organized interests on Einsteinian doctrines, which inconsistencies are so serious to be known nowhere as "catastrophic". The apparent resolution of the inconsistencies will be presented in Chapters 3, 4, 5, 13, and 14.

Let us begin with the following basic requirement for any classical theory of gravitation to be consistent:

**REQUIREMENT 1.** Any consistent classical theory of antimatter must allow a consistent representation of the gravitational field of antimatter. General Relativity does not verify this first requirement because, in order to attempt a compatibility of classical and quantum formulations, antimatter requires negative-definite energies, while general relativity solely admit positive-definite energies, as well known.

Even assuming that this insufficiency is somewhat bypassed, general relativity can only represent antimatter via the reversal of the sign of the charge. But the most important astrophysical bodies expected to be made up of antimatter are neutral. This confirms the structural inability of general relativity to represent antimatter in a credible way.

**REQUIREMENT 2.** Any consistent classical theory of antimatter must be able to represent interior gravitational problems. General relativity fails to verify this second requirement for numerous reasons, such as the inability to represent the density of the body considered, an irreversible condition, e.g., due to the increase of entropy, the locally surging speed of light, etc.

**REQUIREMENT 3.** Any consistent classical theory of gravitation must permit a grand unification with other interactions. It is safe to state that this requirement too is not met by general relativity since all attempts to achieve a grand unification have failed to date since Einstein times (see Chapter 12 for details).

**REQUIREMENT 4.** Any consistent classical theory of gravitation must permit a consistent operator formulation of gravity. This requirement too has not been met by general relativity, since its operator image, known as quantum gravity [18], is afflicted by additional independent inconsistencies mostly originating from its unitary structure as studied in the next section.

**REQUIREMENT 5.** Any consistent classical theory of gravitation must permit the representation of the locally surging nature of the speed of light. This requirement too is clearly violated by general relativity.

\[
G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \cdot R/2 = 0,
\]  

where, as a central condition to have Einstein's gravitation, there are no sources for the exterior gravitational field in vacuo for a body with null total electromagnetic field (null total charge and magnetic moment).

For our needs, we define as general relativistic any description of gravity on a Riemannian space over the realm with Einstein-Hilbert field equations with a

\[ x^4 = x^2; \quad x^2 = k \]
Proof. Quantum electrodynamics has established that the mass of all elementary particles, whether charged or neutral, has a primary electromagnetic origin, that is, all masses are of first-order origin given by the volume integral of the 00-component of the energy-momentum tensor \( T_{\mu\nu} \) of electromagnetic origin, that is, for stars with null total charge.

For the interior problem of the star *s* we have the additional presence of short range weak and strong interactions representable with a new tensor \( T_{\gamma\nu} \).

Therefore, quantum electrodynamics requires the presence of a first-order source tensor in the exterior field space, as in Eqs. (1.4.2). Such a source tensor is absent in Einstein’s gravitation (1.4.1) by conception. Consequently, Einstein’s gravitation is incompatible with quantum electrodynamics. The incompatibility of general relativity with quantum electrodynamics is established by the fact that in the Eqs. (1.4.2) of higher order in magnitude, the electromagnetic force has been equated with the gravitational field, while according to quantum electrodynamics said source is of first order, thus not being measurable in first approximation.

The inconsistency of both Einstein’s gravitation and general relativity is finally established by the fact that, for the case when the total charge and magnetic moment of the body considered are null, Einstein’s gravitation and general relativity allows no source at all. By contrast, as illustrated in Ref. [21], quantum electrodynamics requires a first-order source tensor even when the total charge and magnetic moment of matter are null due to the charge structure of matter.

The first consequences of the above property can be expressed via the following:

**COROLLARY 1.4.1A [21]**: Einstein’s gravitation of vacuum in any curved space without source is incompatible with physical reality.

A few comments are now in order. As is well known, the mass of the electron is entirely of electromagnetic origin, as described by Eq. (3.5), therefore requiring a first-order source term in vacuum as in Eq. (1.4.2). Therefore, Einstein’s gravitation for the case of the electron is inconsistent with nature. Note, however, there is no first-order problem at all, in which case the gravitational and inertial masses coincide.

Next, Ref. [21] proved Theorem 1.4.1 for the case of a neutral particle by showing that the star mass also needs a first-order source tensor in the exterior gravitational problem in vacuum, thus not being ignorable in first approximation.

**COROLLARY 1.4.1B [21]**: In order to achieve compatibility with electromagnetic, weak and strong interactions, any gravitational theory must admit two source tensors, a traceless tensor for the representation of the electromagnetic origin of mass in the exterior gravitational problem, and a second tensor to represent the contributions to interior gravitation of the short range interactions according to the field equations

\[
G_{\mu\nu}^{\text{ShortRange}} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{2}{3} \kappa (\pi_{\mu\nu} + \Pi^{\text{sources}}_{\mu\nu}).
\]

A main difference of the two source tensors is that the electromagnetic tensor \( G_{\mu\nu}^{\text{EM}} \) is notoriously traceless, while the second tensor \( G_{\mu\nu}^{\text{ShortRange}} \) is not. A more rigorous definition of these two tensors will be given shortly.

It should be indicated that, for a possible solution of Eqs. (1.4.6), various explicit forms of the electromagnetic fields as well as of the short range fields originating the electromagnetic and short range energy momentum tensors are given in Ref. [21]. Since both source tensors are positive-definite, Ref. [21] concluded that the interior gravitational problem characterizes the inertial mass according to the expression

\[
\mu_{\text{inertial}} = \int d^4x \sqrt{-g} \pi_{\text{inertial}}.
\]

where the equality solely applies for the electron. Finally, Ref. [21] proved Theorem 1.4.1 for the exterior gravitational problem of a neutral massive body, such as the electron, by showing that the situation is essentially the same as that for the electron. The sole difference is that the electromagnetic field requires the sum of the contributions from all elementary constituents of the star.

By noting that trace terms can be transferred from one tensor to the other in the r.h.s. of Eqs. (1.4.10), it is easy to prove the following:

**COROLLARY 1.4.1A [21]**: Except for possible factorization of common terms, the f- and r-tensors of Theorem 3.2 coincide with the electromagnetic and short range tensors, respectively, of Corollary 1.4.1B.

A few historical comments regarding the Freud identity are in order. It has been pointed out that the Freud identity possesses only four identities (see, e.g., Ref. [17]). While Freud’s identity is that the gravitational mass (1.4.1) is the same as the inertial mass, the relativistic solution gives the gravitational mass (1.4.1) to be different from the inertial mass, as pointed out by Ref. [22].

Finally, Ref. [21] proved Theorem 1.4.1 for the exterior gravitational problem of a neutral massive body, such as a star, by showing that the situation is essentially the same as that for the electron. The sole difference is that the electromagnetic field requires the sum of the contributions from all elementary constituents of the star.
fact, the Freud identity did not escape Pauli who quoted it in a footnote of his celebrated book of 1938 [25]. Santilli became aware of the Freud identity via an accurate reading of Pauli’s book (including its important footnotes) and assumed the Freud identity as the geometric foundation of the gravitational studies presented in Ref. [30].

Subsequently, in his capacity as Editor in Chief of Algebra Groups and Geometries, Santilli requested the mathematician Hanno Rund, a known authority in Riemannian geometry [26], to inspect the Freud identity for the scope of in- containing whether the said identity was indeed a new identity. Rund kindly accepted Santilli’s invitation and released paper [26] of 1991 (the last paper prior to his departure) in which Rund confirmed the identity of Eq. (1.10) as a genuine, independent, fifth identity of the Riemannian geometry. The Freud identity was also rediscovered by Yilmaz (see Ref. [27] and papers quoted therein) who asked the identity for his own broadening of Einstein’s grav- itation via an external stress-energy tensor that is essentially equivalent to the source tensor with nonnull trace of Ref. [11a], Eqs. 1.4.4.

Despite these efforts, the presentation of the Freud identity to various meetings and several personal mailings to colleagues in gravitation, the Freud identity continues to remain vastly ignored to the present day, with very rare exceptions (the indication by colleagues of additional studies on the Freud identity not quoted herein would be gratefully appreciated.)

Theorems 1.4.1 and 1.4.2 complete our presentation on the catastrophic incon- sistencies of Einstein’s gravitation due to the lack of a first-order source in the exter- nitional gravitational problem in vacuum. These theorems, by no means, exhaust all inconsistencies of Einstein’s gravitation, and numerous additional inconsistencies do indeed exist.

For instance, Yilmaz [27] has proved that Einstein’s gravitation explains the 43" of the precession of Mercury, but cannot explain the basic Newtonian con- tribution. This result can also be seen from Ref. [21] because the lack of source and base in Einstein’s gravitation, and numerous additional inconsistencies do indeed exist.

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Figure 1.10. A consistent resolution of the reason the author was unable to accept “Einstein’s gravitation” as a correct theory since the time of his high school studies, the free fall of bodies under gravity that he once mistakenly assumed to a straight radial line, thus without any possible divergence of the free fall along a curved path. This is a result that is far above the limits for an acceptable theory not only to incorporate the NONNEWTONIAN attraction in a clear and unambiguous way, but also in the way that all contributions from curvature should disappear for the free fall in favor of the pure Newtonian attraction. The fact that evidence as incontrovertible continues to be in direct opposition to the Einsteinian theories and their followers, most holding the basis of high school studies, confuses the evidence of a scientific observation of potentially historical properties.

4.4 Catastrophic Inconsistencies of General Relativity due to Curvature

We now turn to the study of the structural inconsistencies of general relativity caused by the very use of the Riemannian curvature, irrespective of the selected field equations, including those fully compatible with the Freud identity.

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In conclusion, not only is Einstein’s reduction of gravity to pure curvature in- consistent with nature because of the lack of sources, but also all the inconsistencies rest in the curvature itself when assumed for the represen-

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THEOREM 4.4.3 [26]: Gravitational theories on a Riemannian space over a field of real numbers do not possess time invariant base units and numerical predictions, thus having serious mathematical and physical inconsistencies.

Proof. The map from Minkowski to Riemannian space is known to be non- canonical.

$$\eta = \text{diag}(1,1,1,-1) \rightarrow \xi(x) = \gamma(x) \eta \gamma(x)^T$$

$$E(x) \neq E(\xi(x))$$

Thus, the time-evolution of Riemannian theories is necessarily noncanonical, with consequential lack of invariance in time of the base units of the theory, such as

$$I_{\eta} = \text{diag}(1,1,1,1,1) \neq I_{\xi(x)}$$

The lack of invariance in time of numerical predictions then follows from the known “covariance”, that is, lack of time invariance of the line element, q.v.d.

As an illustration, suppose that an experimentalist assumes at the initial time $t = 0$ such as the 43" for the precession of the Mercury. One can prove that the same prediction at a later time $t > 0$ is numerically different precisely in view of the “covariance”, rather than invariance as intended in special relativity, thus preventing a serious application of the theory to physical reality. We therefore have the following.

COROLLARY 4.4.4 [26]: Riemannian theories of gravitation in general, and Einstein’s gravitation in particular, can at best describe physical reality at a fixed value of time, without a consistent dynamical evolution.

Interested readers can independently prove the latter occurrence from the lack of covariantization of Einstein’s gravitation. It is known in analytic mechanics (see, e.g., Refs. [17,25]) that Lagrangian theories not admitting an equivalent Hamiltonian counterpart, as is the case for Einstein’s gravitation, are inconsistent under time evolution, unless there are suitable subsidiary constraints that are absent in general relativity.

It should be indicated that the inconsistencies are much deeper than that indicated above. For consistency, the Riemannian geometry must be defined on the field of numbers $\mathbb{R}(\mathbb{C},+,-)$ that, in turn, is fundamentally dependent on the basic unit $I$. But the Riemannian geometry does not leave time invariant the gravitation of gravity, due to its inherent noncanonical character at the classical level with consequential noncanonical structure at the operator level.

Serious mathematical and physical inconsistencies are then unavoidable under these premises, thus establishing the impossibility of any credible use of general relativity, for instance, as an argument against the title time invariance predicted for antimatter in the field of matter [5], as well as establishing the need for a profound revision of our current views on gravitation.

THEOREM 4.4.4. Gravitational experimental measurements do not verify gen- eral relativity uniquely.

Proof. All claimed “experimental verifications” of Einstein’s gravitation are based on the PPN “expansion” (or linearization) of the field equations (such as the post-Newtonian approximation) that, as such, is not unique. In fact, Eqs. (4.4.1) admit a variety of inequivalent expansions depending on the selected parameter, the selected expansion and the selected truncation. It is then easy to show that the selection of an expansion of the same equations (4.4.1) but different from the PPN approximation leads to dramatic departures from gravitational expectations. q.v.d.

THEOREM 4.4.5. General relativity is incompatible with experimental evi- dence because it does not represent the bending of light in a consistent, unique and invariant way.

Proof. Light carries energy, thus being subjected to a bending due to the conventional Newtonian gravitational attraction, while, general relativity pre-dicts that the bending of light is entirely due to curvature (see, e.g., Ref. [17], Section 40.3). In turn, the absence of the Newtonian contribution causes other catastrophic inconsistencies, such as the inability to represent the free fall where curvature does not exist (Theorem 4.4.6 below). Assuming that consistency is achieved with yet unknown manipulations, the representation of the bending of light is not unique because based on a nonunique PPN approximation having different parameters for different expansions. Finally, assuming that consistency and uniqueness are somewhat achieved, the representation is not invariant in time due to the noncanonical structure of general relativity.

THEOREM 4.4.6. General relativity is incompatible with experimental evi- dence because of the lack of consistent, unique and invariant representation of the free fall of test bodies along a straight radial line.

Proof. A consistent representation of the free fall of a test body along a straight radial line requires that the Newtonian attraction be represented by
the field equations necessarily without curvatures, thus depriving the customary belief needed to avoid Cauchy’s 1.4.2.4 A that said Newtonian attraction emerges at the level of the post-Newtonian approximation, q.e.d.

The absence in general relativity at large, thus including Einstein’s gravitation, of well defined contributions due to the Newtonian attraction and to the assumed curvature of space-time, and the general diminution of the former in favor of the latter, creates other catastrophic inconsistencies, such as the inability to represent the base Newtonian contribution in planetary motion as shown by Yilmaz [47], and other inconsistencies [48-52]. A comparison between special and general relativity is here in order. Special relativity can be safely claimed to be “verified by experiments” because the clear identifications of the assumed multiplication will soon make of it the unique and unambiguous version of general relativity, while the admission of the Minkowski spacetime is basically insufficient on dynamical grounds (trivially, because on said tangent space gravitation is absent).

The origin of such a drastic difference is due to the fact that the noncanonical predictions of special relativity are rigorously controlled by the basic Poincaré covariance [10]. By contrast, one of the several drawbacks of the “covariance” of general relativity is precisely the impossibility of predicting numerical values in a unique and unambiguous way, thus preventing serious claims of true "experimental verifications" of general relativity.

The reader should keep in mind the additional well known inconsistencies of the field theory and the catastrophic axiomatic inconsistencies due to lack of invariance [11m], time has indeed arrived for the scientific community to admit the origin of such a drastic difference is due to the fact that the noncanonical predictions of special relativity are rigorously controlled by the basic Poincaré covariance [10]. By contrast, one of the several drawbacks of the “covariance” of general relativity is precisely the impossibility of predicting numerical values in a unique and unambiguous way, thus preventing serious claims of true "experimental verifications" of general relativity.

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When facing the limitations of special relativity and quantum mechanics for the representation of extended, nonspherical, deformable and hyperdense particulars and antiparticles under linear and nonlinear, local and nonlocal as well as a potential and nonpotential forces, a rather general attitude is that of attempting their generalization via the broadening into noncanonical and nonunitary structures, while preserving the mathematics of their original formulation.

Despite the widespread publication of papers on theories with noncanonical or nonunitary structures in refereed journals, including those of major physical societies, it is not generally known that these broader theories are afflicted by inconsistencies so serious to be also known as catastrophic.

Another basic objective of this monograph is the detailed identification of these inconsistencies because their only known resolution is that presented in the next chapters, that permitted by new mathematics specifically constructed from the physical conditions considered.

In fact, the broadening of special relativity and quantum mechanics into noncanonical and nonunitary forms, respectively, is necessary to exit from the class of equivalence of the conventional formulations. The resolution of the catastrophic inconsistencies of these broader formulations when treated via the mathematics of canonical and unitary theories, then leaves no other possibility than that of broadening the basic mathematics.

To complete the presentation of the formulations of the covering hadronic mechanics, in the next sections we shall review the inconsistencies of noncanonical and nonunitary theories. The remaining sections of this chapter are devoted to an outline of hadronic mechanics so as to allow the reader to enter in a progressive way into the advanced formulations presented in the next chapters.

1.5.2 Catastrophic Inconsistencies of Noncanonical Theories

As recalled in Section 1.2, the research in classical mechanics of the 20-th century has been dominated by Hamiltonian systems, that is, systems admitting their complete representation via the truncated Hamilton equations (1.2.2), namely, the historical equations proposed by Hamilton in which the external terms have been cut out.
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These transformations always exist under the necessary continuity and regularity conditions, as guaranteed by Euler-Koening's theorem of analytic mechanics or the Durrer-Bohm Theory of the symmetric gauge [96].

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The representation of open-nonconservative non-Hamiltonian systems required the identification of yet another brackets with a consistent algebra in the brackets of the time evolution, yet such that the basic brackets are not anti-symmetric. The solution was reached in monographs [38] via the Birkhoff-Summable mechanics with basic analytic equations

\[
\frac{d\theta}{dt} = \omega \times \frac{dH(k, l, \ldots)}{dt} + P''(k, l, \ldots) \times \frac{dH(k, l, \ldots)}{dt} \equiv S''(l, k, \ldots) \times \frac{dH(k, l, \ldots)}{dt}.
\]

(1.5.14)

where the tensor \( S'' \) is Summable according to Santilli's [39] realization of Albert's [30] abstract formulation, namely, in the sense that the generalized brackets of the time evolution

\[
\frac{dA}{dt} = (A, B) \times \frac{dA}{dt} + S''(A, B) \times \frac{dA}{dt} \equiv S''(A, B) \times \frac{dA}{dt}.
\]

(1.5.15)

verify all conditions to characterize an algebra, and their attached antisymmetric brackets

\[
[A, B]'' = (A, B) - (B, A).
\]

(1.5.16)

characterize a generalized Lie algebra as occurring in Birkhoffian mechanics. The representation of the open-nonconservative character of the equations was then consequent, since the lack of antisymmetry of the brackets yields the correct time rate of variation of the energy \( E = B \times \frac{dE}{dt} = E \times \frac{dE}{dt} \)

(1.5.17)

and the same occurs for all other physical quantities. Monographs [38] then showed that the Birkhoffian-Summable mechanics for all open-nonconservative systems, identified its transformation theory and provided the following elementary, yet universal realization of the Summable tensor \( S \) for \( B = H \) representing the total nonconserved energy

\[
S'' = \left[ I - \frac{1}{H} \frac{dH}{d\theta} \right] \frac{dH}{d\theta}.
\]

(1.5.18)

Note that the Birkhoffian-Summable mechanics is structurally irreversible, in the sense of being irreversible for all possible energies and Birkhoffian functions since the basin Lie-admissible tensor is itself irreversible, \( \delta[S] \neq S - \delta[A] \), thus being particularly suited to represent irreversible systems.

However, studies conducted after the publication of monographs [9,38] revealed the following seemingly innocuous feature:

**LEMMA 1.5.1 [11b]:** Birkhoffian and Birkhoff-Summable mechanics are non-canonical, i.e., the generalized canonical transformations, are non-conserved,

\[
\omega \neq \frac{d\theta}{dt} = \frac{dH}{d\theta} \times \frac{dH}{d\theta} \neq \omega \times \frac{dH}{d\theta} \times \frac{dH}{d\theta}.
\]

(1.5.19)

The generalized Lie tensor \( \omega \) is of course the type of \( \omega \times \frac{dH}{d\theta} \times \frac{dH}{d\theta} \), required to be a Lorentzian structure, and the generalized Lie tensor represents the total energy, and the Hamiltonian \( H \) which is the reduced sum of an open system, in the sense that the total energy is not only of potential type, represented with \( \omega \times \frac{dH}{d\theta} \times \frac{dH}{d\theta} \), as in the case of the conservative systems, and partly of nonpotential type, thus requiring a constant bracket in the sense of irreversible for all possible energies and Birkhoffian functions such as Jupiter.

Theorem 1.1.3 [11b] then states that the generalized Lie tensor represents the total energy, and the Hamiltonian \( H \) which is the reduced sum of an open system, in the sense that the total energy is not only of potential type, represented with \( \omega \times \frac{dH}{d\theta} \times \frac{dH}{d\theta} \), as in the case of the conservative systems, and partly of nonpotential type, thus requiring a constant bracket in the sense of irreversible for all possible energies and Birkhoffian functions such as Jupiter.

These theorems always exist under the necessary continuity and regularity conditions, as guaranteed by Euler-Koening's theorem of analytic mechanics or the Durrer-Bohm Theory of the symmetric gauge [96].

This first attempt has no physical value because of excessive problems identified in Section 1.2, such as the lack of physical meaning of quantum formulations in the sense that the impossibility of placing a measuring apparatus in the transformed coordinates, the loss of all known relativities due to the necessary nonlinearity of the transforations with consequent mapping of inertial into noninertial frame, and other problems.

The above problems force the restriction of analytic representations of non-Hamiltonian systems within the fixed coordinato of the experimenter, the so-called direct analytic representations of Assumption 1.2.4.9]

Under the latter restriction, the second logical attempt for quantitative treatments of non-Hamiltonian systems is that of broadening conventional canonical theories into a noncanonical form at least admitting a consistent algebra in the brackets of the time evolution, even though the resulting time evolution of the broader equations cannot characterize a canonical transformation.

As an illustration of these second lines of research, in 1978 the author wrote for Springer-Verlag his first volume of Foundations of Theoretical Mechanics [9a] devoted to the integrability conditions for the existence of a Hamiltonian representation (the so-called Belohoroj's conditions of variational selfadjointness).

The evident scope was that of identifying the limits of applicability of the theory within the fixed coordinates of the experimenter.

A main result was the proof that the truncated Hamilton equations admit a direct analytic representation in three space dimensions only of systems with potential (variationally selfadjoint) forces, thus representing only a small part of what are generally referred to as Newtonian systems.

In this way, monograph [9a] confirmed the need to enlarge Hamiltonian mechanics within the fixed frame of the experiment in such a way to admit a direct representation of all possible Newtonian systems verifying the needed regularity and continuity conditions.

Along the latter line of research, in 1982 the author published with Springer-Verlag his second volume of Foundations of Theoretical Mechanics [9b] for the specifically stated objective of broadening conventional Hamiltonian mechanics in such a way to achieve direct universality, that is, the capability of representing all Newtonian systems (universality) in the fixed frame of the experimenter (direct universality), while jointly preserving not only an algebra, but actually the Lie algebra in the brackets of the time evolution.

**LEMMA 1.5.1 [11b]:** Birkhoffian and Birkhoff-Summable mechanics are non-canonical, i.e., the generalized canonical transformations, are non-conserved,

\[
\omega \neq \frac{d\theta}{dt} = \frac{dH}{d\theta} \times \frac{dH}{d\theta} \neq \omega \times \frac{dH}{d\theta} \times \frac{dH}{d\theta}.
\]

(1.5.19)
It is important to understand that Birkhoff and Birkhoff-admissible mechanics are mathematically attractive, but they are not recommended for physical applications, both classically as well as foundations of operator theories.

The canonical Lie tensors has the well known explicit form (1.5.2). Therefore, the diagonal matrix $I_{1.2}$ is left invariant by canonical transformations. But $I_{1.2}$ is the fundamental unit of the basic Euclidean geometry. As such, it represents in an abstract and dimensionless form the basic units of measurement, such as $I_{1.2} = \text{diag}(1, 1, 1, 1)$. (1.5.20)

By their very definition, noncanonical transformations do not preserve the basic unit, namely, they are transformations of the representative type (with arbitrary new values)

$$I_{1.2} = \text{diag}(1, 1, 1, 1)$$

$$U = I_{1.2} \neq I, \quad U \neq I$$

(1.5.21a) (1.5.21b)

where $U$ stands for transposed. We, therefore, have the following important:

**THEOREM 1.5.1**: Whether Lie or bio-admissible, all classical noncanonical theories are affected by catastrophic mathematical and physical inconsistencies.

Proof: Noncanonical theories do not leave invariant under time evolution the basic unit. This implies the loss under the time evolution of the basic field on which the theory is defined. Still in turn, the loss in time of the basic field implies catastrophic mathematical inconsistencies, such as the lack of preservation in time of metric spaces, geometries, symmetries, etc., since the latter are defined over the field of real numbers.

Similarly, noncanonical theories do not leave invariant under time evolution the basic units of measurements, thus being inapplicable for consistent measurements.

The same noncanonical theories also do not possess invariant numerical predictions, thus suffering catastrophic physical inconsistencies, e.g.,

In conclusion, the requiring of a consistent algebra in the brackets of the time evolution, as it is the case for Birkhoff and Birkhoff-admissible mechanics, is not sufficient for consistent physical applications because the theories remain noncanonical. In order to achieve a physically consistent representation of non-Hamiltonian systems, it is necessary that

1) The analytic equations must be derivable from a first-order variational principle, as necessary for quantization.

2) The brackets of the time evolution must characterize a consistent algebra admitting exponentiation to a transformation group, as necessary to formulate symmetries and

that can be written explicitly in the familiar forms

$$\frac{\partial A}{\partial t} + H = 0$$

(1.4.26a)

$$\frac{\partial A}{\partial r} - p_t = 0$$

(1.5.26b)

The use of the same quantization

$$\mathcal{A} \to -i \hbar \times \partial \psi$$

(1.5.27)

yields Schrödinger’s equations in a unique and unambiguous way

$$\frac{\partial \mathcal{A}}{\partial t} + H = 0 \to -i \times \frac{\partial \psi}{\partial \mathcal{A}} - H \times \psi = 0$$

(1.5.28a)

$$\frac{\partial \mathcal{A}}{\partial r} = p_t - i \hbar \times \partial \psi = 0$$

(1.5.28b)

$$\frac{\partial \mathcal{A}}{\partial t} = 0 - \frac{\partial \psi}{\partial t} = 0$$

(1.4.28)

The much more rigorous symplectic quantization yields exactly the same results and, as such, it is not necessary for these introductory notes.

A feature crucial for quantization is Eq. (1.5.26b) from which it follows that the canonical action $A^c$ is independent from the linear momentum, i.e.,

$$A^c = A^c(t, r)$$

(1.5.29)

an occurrence generally (but not universally) referred in the literature as characterizing a first-order action functional.

From the naive quantization it follows that, in the configuration representation, the wave function originating from first-order action functionals is independent from the linear momentum (and, vice-versa, in the momentum representation it is independent from the coordinates),

$$\psi = \psi(t, r)$$

(1.5.30)

which property is crucial for the axiomatic structure of quantum mechanics, e.g., for the correct formulation of Rosenbaum’s uncertainty principle, causality, Bell’s inequalities, etc.

A serious knowledge of hadronic mechanics requires the understanding of the reason Birkhoffian mechanics cannot be assumed as a suitable foundations for
A first problem is that the latter equations are generally nonlinear and, as such, they cannot be generally solved in the r- and p-coordinates. This causes the emergence of an operator mechanics in which it is impossible to define basic physical quantities, such as the linear momentum or the angular momentum, with consequent lack of currently known physical relevance at this moment.

On more technical grounds, in the lifting of Hamiltonian into birhdonic mechanics, there is the replacement of the r-coordinates with the B-functions. In fact, the birhdonic action has the explicit dependence on the B-functions, $A = A(B)$, $A' = A'(B)$. As such, a birhdonic action can indeed be interpreted as being of first-order, but not in the $r$-functions, rather in the $B$-coordinates.

Consequently, a correct operator image of the birhdonic mechanics is given by the expressions (first derived in Ref. 151) 

$$\tag{1.5.3a} x \times \vartheta (x, B)(x) = B \vartheta (x, B)(x),$$

$$\tag{1.5.3b} \vartheta (x, B)(x) = \vartheta (x, B)(x),$$

As we shall see in Chapter 3, the above equations characterize a covering of hadronic (rather than quantum) mechanics, in the sense of being structurally more general, yet admitting hadronic mechanics as a particular case.

Even though mathematically impeccable, intriguing, and deserving further studies, the mechanics characterized by Eqs. (1.5.30) is excessively general for our needs, and its study will be left to the interested reader.

The above difficulties identify quite precisely the first basic problem for the achievement of a physically consistent and effectual formulation of hadronic mechanics, consisting in the need of constructing a new mathematics capable of representing CLOSED (that is, isolated) non-Hamiltonian systems via a first-order variational principle (as required for consistent quantitation), admitting antisymmetric brackets in the time evolution (as required by conservation laws), and possessing time invariant units and numerical predictions (as required for physical values).

The need to construct a new mathematics is evident from the fact that no pre-existing mathematics can fulfill the indicated needs. As we shall see in Chapter 3, Santilli's nonunitary mathematics [38] has been constructed precisely for and does indeed solve those specific problems.

The impossibility of assuming the Birkhoff-admissible mechanics as the foundation of operator formulation for OPEN (that is, nonconservative) non-Hamiltonian systems is evident from the fact that no pre-existing mathematics can fulfill the indicated needs. As we shall see in Chapter 3, Santilli's nonunitary mathematics [38] has been constructed precisely for and does indeed solve those specific problems.

The author has dedicated his research life to the construction of axiomatically consistent and irrealist generalizations of quantum mechanics for the treatment of nonlinear, nonlocal, and nonunitary effects because they are crucial for the prediction and treatment of new clean states and forces.

In this section we review the foundations of these studies with the identification, most importantly, of the failed attempts in the hope of assisting receptive studies, the mechanics characterized by Eqs. (1.5.39) is excessively general for our needs, and its study will be left to the interested reader. In the above reference, the reader should be aware that these exist in the literature numerous classes of "generalizations of quantum mechanics" that claim to have a physical relevance, which are not generalizations in the sense of being structurally more general, yet admitting hadronic mechanics as a particular case.

A consequence has been the widespread belief that nonpotential interactions "do not exist" in the particle world, a view based on the lack of existence of their mathematical formulation of contact, nonconservative and nonpotential interactions.

As a consequence, the resolution of the difficulties in the quantitation of nonpotential interactions achieved by hadronic mechanics implies a rather profound revision of most of the scientific views of the 20th century, as we shall see in the subsequent chapters.

\subsection{1.5.3 Catastrophic Inconsistencies of Nonunitary Theories}

The limitations of quantum mechanics are understood (and admitted), another natural tendency is to exit from the class of equivalence of the theory via suitable generalizations, while keeping the mathematical methods used for quantum mechanics. It is for these studies to understand that these efforts are afflicted by catastrophic mathematical and physical inconsistencies, and to those so afflicted by classical noncanonical formulations based on the mathematics of canonical theories.

\begin{align}
\tag{1.5.13}
\mathcal{A} & = \mathcal{A} \times \mathcal{B} \times \mathcal{R} \\
\mathcal{B} & = \mathcal{B} \times \mathcal{C} \\
\mathcal{R} & = \mathcal{R} \times \mathcal{S}
\end{align}
By remembering that the Lie product characterizes Blochmann’s equations, the above generalized product was submitted as part of the following parametric generalization of Heisenberg’s equations in its finite and infinite forms [41,42]

\[
\begin{align*}
\mathbf{A}(\mathbf{t}) &= \mathbf{A}(\mathbf{0}) \times \mathbf{U}(\mathbf{t})^\dagger = e^{i\mathbf{H} \mathbf{t}} \mathbf{A}(\mathbf{0}) \times e^{-i\mathbf{H} \mathbf{t}} \mathbf{U}(\mathbf{t})^\dagger \times \mathbf{A}(\mathbf{0}) \\
\mathbf{i} \mathbf{A} \mathbf{d}t &= [\mathbf{A}, \mathbf{H}] = \mathbf{p} \times \mathbf{A} \times \mathbf{H} \times \mathbf{B} \times \mathbf{A}, \\
\end{align*}
\]

(1.5.43a)

with classical counterpart studied in Ref. [41].

After an extensive research in European mathematical literature (conducted prior to the publication of Ref. [41] with the results listed in the same publication), the brackets (\(\mathbf{A}, \mathbf{B}\)) = \(\mathbf{p} \times \mathbf{A} \times \mathbf{B} \times \mathbf{B} \times \mathbf{A}\) were to be Lie-admissible according to A. A. Albert [40], that is, the brackets are such that their attached antisymmetric product

\[
[A;B] = [A,B] - [B,A] = (\mathbf{p} \times \mathbf{A} \times [A,B]),
\]

(1.5.44)

characterizes a Lie algebra.

Jointly, brackets (\(\mathbf{A}, \mathbf{B}\)) are Jordan admissible also according to Albert, in the sense that their attached symmetric product,

\[
[A;B] = [A,B] + [B,A] = (\mathbf{p} \times \mathbf{A} \times [A,B]),
\]

(1.5.45)

characterizes a Jordan algebra.

At that time (1967), only three articles on this subject had appeared in Lie- and Jordan-admissible in the sole mathematical literature (see Ref. [35]).

In 1985, Biederharn [44] and MacFarlane [45] published their papers on the simpler q-deformations \(A;B = \mathbf{p} \times A \times B \times B \times A\) without a quotation of the origination of the broader form \(A;B = \mathbf{p} \times A \times B \times B \times A\) by Santilli [41] in 1967.

Evidently, Biederharn and MacFarlane obtained from quoting Santilli’s origination of twenty years earlier despite their documented knowledge of such an origination.

For instance, Biedenharn and Santilli had applied for a DOE grant precisely on the same deformations two years prior to Biedenharn’s paper of 1985, and Santilli had personally informed MacFarlane of said deformations years before his paper of 1985.

The lack of quotation of Santilli’s origination of q-deformations resulted in a large number of subsequent papers by numerous other authors that also obtained from quoting and origination (see representative contributions [46-49], for which

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is formulated via the multi-valued hyperstructures of Chapter 5, Eqs. (5.3).
The above rules confirm the preservation of a Lie-admissible structure under the most general possible transforms, thus confirming the direct universality of laws (1.4.49) as per Theorem 1.4.2. The point is that the formulations are not invariant because

\[ P' = (U \times U^{-1}) \times (U \times Q \times U^{-1}) \times (U \times U^{-1})' \neq P, \]

\[ Q' = (U \times U^{-1}) \times (U \times Q \times U^{-1}) \times (U \times U^{-1})' \neq Q, \]

that is, because the product itself is not invariant.

By comparison, the invariance of quantum mechanics follows from the fact that the associative product "v" is not changed by unitary transforms

\[ U \times U^{-1} \times U \times U^{-1} = U, \]

\[ (A \times B) \times (A \times B) = A \times B, \]

Therefore, generalized Lie-admissible and Jordan-admissible theories (1.5.49) are not invariant because the generalized products "v" and "r" are changed by nonunitary transformations, including the time evolution of the theory itself. The same result also holds for other nonunitary theories, as the reader is encouraged to verify.

The mathematical inconsistencies of nonunitary theories are the same as those of nonassociative theories. Recall that mathematics is formulated over a given field of numbers. Whenever the theory is nonunitary, the first noninvariance is that of the basic unit of the field.

The lack of conservation of the unit then causes the loss of the basic field of numbers on which mathematics is constructed. It then follows that the entire axiomatic structure as formulated at the initial time, is no longer applicable at subsequent times.

For instance, the formulation of a nonunitary theory on a conventional Hilbert space has no mathematical sense because that space is defined over the field of complex numbers.

The loss of the latter property under nonunitary transforms then implies the loss of the former. The same result holds for metric spaces and other mathematics based on a field.

In short, the lack of invariance of the fundamental unit under nonunitary time evolutions causes the catastrophic collapse of the entire mathematical structure, without known exception.

The reader should be aware that the above physical and mathematical inconsistencies apply not only for Eqs. (1.4.49) but also for a large number of general structures, as expected from the direct universality of the formalism.

It is of the essence to identify in the following at least the most representative cases of physically inconsistent theories, to prevent their possible application (see Ref. [36] for details):

1) Dissipative nuclear theories [13] represented via an imaginary potential in non-Hermitian Hamiltonians,

\[ H = H_{\text{Re}} \neq H^* \]  

lose all algebra in the brackets of their time evolution (requiring a bilinear product) in favor of the triple system,

\[ \varepsilon \times (A \times B) = A \times B + H_{\text{Re}} \neq A \times [A, B, H_{\text{Re}}]. \]  

(1.5.50)

This causes the loss of nuclear notions such as "protons and neutrons" as conventionally understood, e.g., because the definition of their spin mandates the presence of a constant algebra in the brackets of the time evolution.

2) Statistical theories with an external collision term \( C \) (see Ref. [59] and literature quoted therein) and equation of the density

\[ \varepsilon \delta_{\text{dil}} \equiv \rho \rightarrow H = [\rho, B] + C, \quad H = H^* \]  

(1.5.60)
Theories the reader can easily identify from the departures of their time evolution from the unitary line.

All the above theories have a nonunitary structure formulated via conventional mathematics and, therefore, are afflicted by the catastrophic physical and mathematical inconsistencies of Theorem 1.5.2.

Additional generalized theories were attempted via the relaxation of the linear character of quantum mechanics [56]. These theories are essentially based on eigenvalue equations with the structure

$$H(x, y, p, n)(\psi) = E(\psi) \text{ } \psi$$

(1.5.65)

where \( H \) depends on the wavefunction.

Even though mathematically intriguing and possessing a seemingly unitary time evolution, these theories also possess rather serious physical drawbacks, such as: they violate the superposition principle necessary for composite systems such as a hadron; they violate the fundamental Mocky imprimitivity theorem necessary for the applicability of Galileo’s and Einstein’s relativities and possess other drawbacks [36] so serious to prevent consistent applications.

Yet another type of broader theory is Weinberg’s nonlinear theory [79], with brackets of the type

$$A \star B \star \cdots \star Y = \frac{\partial A}{\partial t} + \frac{\partial A}{\partial x} + \frac{\partial A}{\partial y} + \cdots$$

(1.5.66)

where the product \( A \star B \star \cdots \star Y \) is nonassociative.

The theory violates Olkin’s Non-quantum Theorem [78], prohibiting the use of nonassociative envelopes because of catastrophic physical consequences, such as the loss of equivalence between the Schrödinger and Brauer-Ziegler representations (the former remain associative, while the latter becomes nonassociative, thus resulting in inequivalence).

Weinberg’s theory also suffers from the absence of any unit at all, with consequent inability to apply the theory to measurements, the loss of exponentiation to a finite transform (i.e., lack of Pancchi-Birkoff-Witt theorem), and other inconsistencies studied in Ref. [79].

These inconsistencies are not resolved by the adaptation of Weinberg’s theory proposed by Jordan [80] as readers seriously interested in avoiding the publication of theories known to be inconsistent could be encouraged to study.

Several authors also attempted the relaxation of the local-differential character of quantum mechanics via the addition of “integral potentials” in the Hamiltonian,

$$V = \int \text{ } \text{d}x \text{ } \text{d}y \text{ } \text{d}z \text{ } \cdots$$

(1.5.67)

These theories are structured on both mathematical and physical grounds.

**3. The new brackets must be anti-symmetric in order to allow the conservation of the total energy under contact nonpotential internal interactions**

For the case of open, classical or operator irreversable interior systems of particles there is the need of a second generalization of Lie’s theory characterizing broader brackets, hereon denoted \( \{ A, B \} \) verifying the following conditions:

1. The broader brackets \( \{ A, B \} \) must also verify the scalar and distributive laws (3.9) to characterize an algebra:

$$\{ A, B \} = [ A, B ] = 0; \text{ } (\{ A, B \}) \text{ } \text{d}t = 0$$

(1.5.70)

2. The broader brackets must include two non-Hermann operators, hereon denoted \( \hat{P} \) and \( \hat{Q} \), so as to represent the two directions of time, and the new brackets, denoted \( \rho(\hat{P}, \hat{Q}) \), must be neither antisymmetric nor symmetric: to characterize the time rate of variation of the energy and other quantities,

$$\{ A, B \} \neq 0; \text{ } (\{ A, B \}) \text{ } \text{d}t = 0$$

(1.5.71)

3. The broader brackets must admit the antisymmetric brackets \( [ A, B ] \) and \( [ A, B ] \) as particular cases because conservation laws are particular cases of non-conservation laws.

For the case of closed and open interior systems of antiparticles, it is easy to see that the above generalizations of Lie’s theory will not apply for the same reason that the conventional Lie theory cannot characterize exterior systems of point-like antiparticles at classical level studied in Section 1.1 (due to the existence of only one quantization channel, the operator image of classical treatments of antiparticles can only yield particles with the wrong sign of the charge, and certainly not their charge conjugate).

The above occurrence is a third generalization of Lie’s theory specifically conceived for the representation of closed or open interior systems of antiparticles at all levels of study, from Newton to second quantization. As we shall see, the latter generalization is provided by the isodual map.

In an attempt to resolve the scientific imbalances of the preceding section, when at the Department of Mathematics of Harvard University, Santilli [39,90] proposed in 1978 an anti-quantizing generalization of conventional mechanics verifying conditions 1), 2) and 3), that he subsequently studied in various works (see monographs [90,10.11.39] and quoted literatures).

The new mathematics is today known as Santilli’s isotopic and genotypic mathematics or isomathematics and genomathematics for the reason that the word “isotopic” or the prefix “iso” are used in the Greek meaning of preserving the original atoms, and the word “genotypic” is used in the sense of inducing new atoms.

Proposal [39] for the new isomathematics was centered in the generalization (called lifting) of the conventional, N-dimensional unit, \( I = \text{Diag} \{ 1, 1, \ldots \} \) into

\[
\begin{align*}
\{ A, B \} & = AB - BA = \int \text{ } \text{d}x \text{ } \text{d}y \text{ } \text{d}z \text{ } \cdots \text{; and}
\{ A, B \} & = \int \text{ } \text{d}x \text{ } \text{d}y \text{ } \text{d}z \text{ } \cdots \text{; and}
\end{align*}
\]
In fact, all conventional linear, local and potential interactions can be represented with a conventional Hamiltonian, while the shape and density of the particles and their nonlinear, nonlocal and nonpotential interactions can be represented with Santilli's ansatz via evaluations of the type

$$I = I_{\text{local}} + I_{\text{nonlocal}} \rho_{\text{local}}(\rho_{\text{local}} + \rho_{\text{nonlocal}}) \rho_{\text{nonlocal}}$$

(1.5.76)

where the $\rho_{\text{local}}$, $\rho_{\text{nonlocal}}$, $\rho_{\text{local}}$ allow to represent, for the first time, the actual, extended, nonspatial and deformable shapes of the particles considered (normalized to the volume $v_0 = 1$ for the perfect sphere); $\rho_{\text{local}}$ represents, also for the first time, the density of the interior medium (normalized by the value $v_\text{d} = 1$ for empty space); the function $\Gamma(\rho, \gamma)$ represents the nonlinear character of the interactions; and the integral $\int d^3\rho(\rho_{\text{local}} + \rho_{\text{nonlocal}})$ represents nonlocal interactions due to the overlapping of particles or of their wave packets.

When the mutual distances of the particles are much greater than $10^{-11}$ m = 1 F, the integral in Eq. (1.5.76) is identically null, and all nonlinear and nonlocal effects are null. When, in addition, the particles considered are reduced to point masses in vacuum, all the quantities are equal to 1, generalized unit (1.3.22) recovers the trivial unit, and isomathematics recovers conventional mathematics identically, uniquely and unambiguously. In the same memoir [39], in order to represent irreversibility, Santilli proposed a broader genomathematics based on the following differentiation of the product to the right and to the left with corresponding generalized units

$$A \times B = A \times \hat{P} \times B \times \hat{P} = I^\Gamma = I^\Gamma$$

(1.7.7a)

$$A \times B = A \times \hat{Q} \times B \times \hat{Q} = I^{\hat{\Gamma}} = I^{\hat{\Gamma}}$$

(1.7.7b)

where evidently the product to the right, $A \times B$, represents motion forward in time and that to the left, $A \times \hat{B}$, represents motion backward in time. Since $A \times \hat{B} \neq A \times B$, the latter mathematics represents irreversibility from the most elementary possible axioms.

The latter mathematics was proposed under a broader lifting called "genotopy" in the Greek meaning of inducing new axioms, and it is known today as Santilli's genomathematics, genoalgebras, genogeometries, genotopologies, etc.

It is evident that genomathematics (1.7.7) require a step by step generalization of all aspects of isomathematics, resulting in granometries, granyield, granyeform, genoalgebras, genogeometries, genoprocesses, etc. [39, 108, 112, 36a].

Via the use of the latter mathematics, Santilli proposed also in the original memoir [39] a genotopy of the main branches of Lie's theory, including a genotypic broadening of universal enveloping associative algebras, Lie-Santilli isounital isoalgebras, Lie-Santilli isoigroup, isorepresentation theory, etc. and the resulting theory is today known as the Lie-Santilli genelgebra with basic brackets

$$\{a, b\} = a \times b + b \times a$$

(1.5.78)

where the subscripts $\hat{P}$ and $\hat{Q}$ shall be dropped from now on. It should be noted that the main proposal of memoir [39] is genomathematics, while isomathematics is presented as a particular case for (1.5.79)

as we shall see in Chapters 3 and 4, the isomathematical and isogalgebras, isogeometries for the treatment of antiparticles are given by the isogeon image (1.6.1) of the above iso- and geno-mathematics, respectively.

1.5.5 Hadronic Mechanics

Thanks to the prior discovery of isomathematics and genomathematics, in memoir [50] also of 1978 Santilli proposed a generalization of quantum mechanics for closed and open interior systems, respectively, under the name of hadronic mechanics, because hyperdense hadrons, such as protons and neutrons, constitute the most representative (and most difficult) cases of interior dynamical systems.

For the case of closed interior systems of particles, hadronic mechanics is based on the following isosymplectic generalization of Heisenberg's equations [86, Eqs. (1.15.34) and (1.18.4)]:

$$i \frac{d}{dt} = [\mathbf{A}\mathbf{I}, \mathbf{B}] = \mathbf{A}\mathbf{B} - \mathbf{B}\mathbf{A}. \quad (1.5.80)$$

while for the broader case of open interior systems hadronic mechanics is based on the following genotopic generalizations of Heisenberg's equations [86, Eqs. (1.18.16)]:

$$i \frac{d}{dt} = [\mathbf{A}, \mathbf{B}] = \mathbf{A}\mathbf{B} - \mathbf{B}\mathbf{A}. \quad (1.5.81)$$

The isogonal images of Eqs. (1.5.80) and (1.5.81) for antiparticles as well as their individual hyperdynamical significations for biological studies, were added more recently (88).

A rather intense scientific activity followed the original proposal [50], including five Workshops on Isogalilean Formulations held at Harvard University from 1979 to 1982, fifteen Workshops on Hadronic Mechanics, and several formal conferences held in various countries, plus a rather large number of research papers and monographs written by various mathematicians, theoreticians and experimentalists, for an estimated total of some 15,000 pages of research published refereed journals (see the General References on Hadronic Mechanics at the end of this volume).

It should be indicated that, following the original proposal of 1978 [50], maturity on the basic new numbers of hadronic mechanics, the iso-, geno- and hypernumbers and their isousuals was reached only in 1993 [87]; a correct mathematical formulation was reached only in 1986 [80] due to problems that had unceasingly voided for years; and a fully rigorous isophysical formulation was reached only in 1997 for invariant Lie-isosymplectic theories [89] and invariant Lie-isomultivalued theories [90] (see also memoir [80]; see also memoir [81] for a recent review).

The lapse of time between the original proposal of 1978 and the achievement of mathematical and physical maturity illustrates the difficulties to be resolved.

As a result of all these efforts, hadronic mechanics is today a rather diversified discipline conceived and constructed for quantitative treatments of all classical and quantum systems of particles according to Definition 1.1.1 with corresponding isogonal formulations for antiparticles.

It is evident that in the following chapters we can review only the most salient foundations of hadronic mechanics and have to defer the interested reader to the technical literature for brevity.

As of today, hadronic mechanics has experimental verifications and applications in particle physics, nuclear physics, atomic physics, superconductivity, chemistry, biology, astrophysics and cosmology, including numerous industrial applications outlined in monograph [92].

Hadronic mechanics can be classified into sixteen different branches, including:

1. branches of classical treatment of particles with corresponding four branches of operator treatment also of particles, and eight corresponding (classical and operator) treatments of antiparticles.

An effective classification of hadronic mechanics is that done via the main topological features of the assumed basic unit, since the latter characterizes all branches according to

$$I = 1 > 0$$

HAMILTONIAN AND QUANTUM MECHANICS

Used for the description of closed and reversible systems of point-like particles in exterior conditions in vacuum:

$$I^P = 1 < 0$$

ISODUAL HAMILTONIAN AND ISODUAL QUANTUM MECHANICS

Used for the description of closed and reversible systems of point-like antiparticles in exterior conditions in vacuum:

$$I^\hat{P} = -1 > 0$$

HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

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HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

CLASSICAL AND OPERATOR BIOMECHANICS
Used for the description of closed and reversible systems of extended particles in interior conditions:
\[ I(x, v, \ldots) = \mathcal{F}(x, \ldots) \]

ISODUAL CLASSICAL AND OPERATOR BIOMECHANICS
Used for the description of closed and reversible systems of extended antiparticles in interior conditions:
\[ I(x, v, \ldots) = \mathcal{F}(x, \ldots) \]

CLASSICAL AND OPERATOR GENOMECHANICS
Used for the description of closed and irreversible systems of extended particles in interior conditions:
\[ I(x, v, \ldots) = \mathcal{F}(x, \ldots) \]

ISODUAL CLASSICAL AND OPERATOR GENOMECHANICS
Used for the description of closed and irreversible systems of extended antiparticles in interior conditions:
\[ I(x, v, \ldots) = \mathcal{F}(x, \ldots) \]

IN SUMMARY, a serious study of antiparticles requires its study beginning at the classical level and then following at each successive level, exactly as it is the case for particles.

In so doing, the mathematical and physical treatments of antiparticles emerge as being deeply linked to that of particles since, as we shall see, the former are an anti-isomorphic image of the latter.

Above all, a serious study of antiparticles requires the admission of their existence in physical conditions of progressively increasing complexity, that consequently require mathematical and physical methods with an equally increasing complexity, resulting in the various branches depicted in Figure 5.

All in all, young minds of any age will agree that, rather than having reached its clinical climax, the proposed construction of hadronic mechanics following the preceding memoir (10) on the model new mathematics:

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References


E. Fermi, Reviews of Modern Physics, 11, 109 (1939).


Chapter 2

ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.1 ELEMENTS OF ISODUAL MATHEMATICS

2.1.1 Isodual Unit, Isodual Numbers and Isodual Fields

The first comprehensive study of the isodual theory for point-like antiparticles has been presented by the author in monograph [34]. However, it is important to review the most recent formulation of the isodual mathematics in sufficient details to render this monograph self-sufficient.

In this section, we identify only those aspects of isodual mathematics that are essential for the understanding of the physical profiles presented in the subsequent sections of this chapter. We begin with a study of the most fundamental elements of all mathematical and physical formulations, units, numbers and fields, from which all remaining foundations can be uniquely and unambiguously derived via simple compatibility arguments. To avoid unnecessary repetitions, we assume the reader has a knowledge of the basic mathematics used for the classical and operator treatment of matter, including a knowledge of the fields of real, complex and quaternionic numbers. The symbol \( \mathbb{R} \) used in this chapter denotes conventional Hermitian conjugation, namely, transpose plus complex conjugation.

**DEFINITION 2.1.1**: Let \( F = \{ a + n \} \) be a field (of characteristic zero), namely a ring with elements given by real numbers \( a = a, \) complex numbers \( a + c = a, \) or quaternionic numbers \( a + c \). A field \( a + n \) has the following properties:

- Conventional sum \( a + b = a + b \) verifying the associative law
- Conventional product \( a \times b = a \times b \) verifying the associative law
- Conventional distributive law \( a \times (b + c) = a \times b + a \times c \) for \( a, b, c \in F \)

For real numbers we have

- Conventional product \( a \times b = a \times b \) verifying the associative law
- Right distributive law \( a \times (b + c) = a \times b + a \times c \) for \( a, b, c \in F \)

**LEMMA 2.1.1**: The above lemma establishes the property (first identified in Refs. [1]) that

\[
\text{isodual quotient fields } \mathbb{F} \text{ and their isodual images } \mathbb{F} \text{ are anti-isomorphic to each other.}
\]

Lemma 2.1.1 and 2.1.2 illustrate the origin of the name “isodual mathematics.” In fact, to represent antimatter the needed mathematics must be a suitable “dual” of conventional mathematics, while the prefix “iso” is used in its Greek meaning of preserving the original axioms.

It is evident that for real numbers we have

\[
a^2 = -a^2,
\]

while for complex numbers we have

\[
b^2 = (a + i + n)^2 = a + i + n = -b^2,
\]

with a similar formulation for quaternions.

It is also evident that, for consistency, all operations on numbers must be subjected to isoduality when dealing with isodual numbers. This implies the isodual powers

\[
a^2 = a^2, a^4 = a^4, a^8 = a^8, \ldots
\]

(n times, with \( n \) an integer); the isodual square root

\[
\sqrt{a^2} = \sqrt{-a^2}, \sqrt{a^4} = \sqrt{a^4} = a^2, a^{1/2} = a^{1/2} = a^{1/2};
\]

the isodual quotient

\[
\mathbb{F} = a^2/\sqrt{a}, b^2/\sqrt{b}, \ldots
\]

etc.

An important property for the characterization of antimatter is the following:

**LEMMA 2.1.3**: Isodual fields have a negative-definite norm, called isodual norm,

\[
|a|^2 = -|a|^2 = -\langle a|a \rangle < 0,
\]

where \( \langle | | \rangle \) denotes the conventional norm.

For isodual real numbers we therefore have the isodual isomorph

\[
|a|^2 = -|a| < 0,
\]

and for isodual complex numbers we have

\[
|a|^2 = -|a|^2 = -\langle a|a \rangle < 0.
\]

**LEMMA 5.1.4**: All quantities that are positive-definite when referred to positive units and related fields of matter (such as mass, energy, angular momentum, density, temperature, time, etc.) become negative-definite when referred to isodual units and related isodual fields of antimatter.

As recalled Chapter 1, antimatter has been discovered in the negative-energy solutions of Dirac’s equation and they were originally thought to evolve backward in time (Stueckelberg, Feynman, and others, see Refs. [1,2]: 11, 12] of Chapter 1). The possibility of representing antimatter via isodual methods is therefore visible already from these introductory notions.

The main novelty is that the conventional treatment of negative-definite energy and time was (and still is) referred to the conventional unit \( +1 \). This lead to a number of contradictions in the physical behavior of antimatter. By comparison, negative-definite physical quantities of isodual theories are referred to a negative-definite unit \( -1 \). This implies a mathematical and physical equivalence between positive-definite quantities referred to positive-definite units, characterizing matter, and negative-definite quantities referred to negative-definite units, characterizing antimatter. These foundations thus permit a novel characterization of antimatter beginning at the Newtonian level, and then persisting at all subsequent levels.

**DEFINITION 2.1.3**: A quantity is called isodual when it coincides with its isodual.

It is easy to verify that the imaginary unit is isodual because

\[
i^2 = -i = \sqrt{-1} = -\langle i | i \rangle.
\]

This property permits a better understanding of the isoduality of complex numbers that can be written explicitly

\[
e^i = (a + b + c + d)^2 = e^i + i + c^2 + d^2 = -(c + d + b + a).
\]

The above property will be important to prove the equivalence of isoduality and charge conjugation at the operator level. As we shall see, isoduality is a new fundamental view of nature with deep physical implications, not only in classical and quantum mechanics but also in cosmology. For instance we shall see that Dirac’s gamma matrices are isoddual.
thus implying a basically new interpretation of this equation that has remained unmatched for about one century. We shall also see that, when applied to cosmology, insoduality implies equal distribution of matter and antimatter in the universe, with identically null total physical characteristics, such as identically null total time, identically null total mass, etc.

We should also indicate that we have assumed the insoduality of the multiplica-
tion, i.e., \( x \cdot y = - (\cdot x) \cdot (\cdot y) = (-x) \cdot (-y) = xy \), but not that of the sum, \( x + y = x + y \) or \( x + y = x + y \).

This approach may not appear entirely motivated to the mathematically in-
classified reader because fields are irreverent under the above defined insoduality of the sum due to the invariance of the additive unit, \( 0 = 0 \) (although fields are not invariant under the insoduality of the product due to the lack of invariance of the multiplicative unit, \( 1 = -1 \)).

The above decision is motivated by pragmatic, rather than mathematical argu-
ments and, more specifically, for compatibility with the more general models and
formulae, studied in the following chapters. In fact, at the latter branched levels, we have the absence of the invariances of the axiom of a field under these broader liftings of the sum. In turn, the loss of the field axioms comes the consequent inapplicability of the theory for physical applications as currently known, that is, based on "numbers" as range verifying the axioms of a field, thus admitting a right and left, well-defined, multiplicative unit representing the selected units of measurements.

We assume the reader is aware of the emergence here of new numbers, those
with a negative unit, that have no connection with ordinary negative numbers and
are the true foundations of the insodual theory of quaternion.

### 2.1.2 Insodual Functional Analysis

All conventional and special functions and transforms, as well as functional
analysis at large, must be subjected to insoduality for consistent applications,
resulting in the simple, yet unique and significant insodual functional analysis,
studied by Kadomtsev [9], Santilli [6], and others.

We here mention the insodual trigonometric functions

\[
s \sin^d \theta^d = - \sin (-\theta^d), \quad \cos^d \theta^d = \cos (-\theta^d),
\]

with related basic property

\[
s \sin^d \theta^d \cdot s \sin^d \theta^d \cdot s \sin^d \theta^d = 1 = -1.
\]


### 2.1.3 Insodual Differential and Integral Calculus

Contrary to a rather popular belief, the differential calculus is indeed depen-
dent on the assumed unit. This property is not so transparent in the conventional
formulation because the basic unit is the trivial number \( +1 \). However, the de-
pendency of the unit emerges rather forcefully under its generalization.

The insodual differential calculus, first introduced by Santilli in Ref. [5a], is
characterized by the insodual derivatives

\[
x^d \cdot y^d = x^d \cdot y^d = x^d \cdot y^d = -y^d \cdot x^d,
\]

with corresponding insodual derivatives

\[
\frac{dx^d}{dy^d} = \frac{-dy^d}{dx^d}, \quad \frac{dx^d}{dy^d} = \frac{-dy^d}{dx^d}.
\]

and related insodual properties. Note that conventional differentials are undefined, i.e.,

\[
\left[ \frac{dx^d}{dy^d} \right] = \frac{dx^d}{dy^d} \cdot \frac{dy^d}{dx^d} = 0.
\]

but derivatives are not undefined.

\[
\left[ \frac{dx^d}{dy^d} \right] = \frac{dx^d}{dy^d} \cdot \frac{dy^d}{dx^d} = 0.
\]

The above properties explain why the insodual differential calculus remains
undiscovered for centuries.

Other notions, such as the insodual integral calculus, can be easily derived and
shall be assumed as known herein.

### 2.1.4 Lie-Santilli Insodual Theory

Let \( L \) be an \( n \)-dimensional Lie algebra in its regular representation with uni-
versal enveloping associative algebra \((L), \overline{(L)} \in L, \) \( n \)-dimensional unit \( I \) =
\( \text{Diag}(1, \ldots, 1, 1) \), ordered set of Hermitian generators \( X = X^d = (X_k) \),
\( k = 1, 2, \ldots, n \), conventional associative product \( X \cdot X \), and familiar Lie
Theorems over a field \( F(a, +) \).

The Lie-Santilli insodual theory was first submitted in Ref. [2] and then studied in
Refs. [4-7] as well as by other authors [23-31].

The insodual conventional and special function in the insodual Euclidean space
\( \mathbb{E}^n(\mathbb{R}) \), defined by

\[
\mathbb{E}^n(\mathbb{R}) = \mathbb{R}^n \times \mathbb{R}_d,
\]

and related insodual

\[
(x - y)^d = (x - y)^d \cdot (x - y)^d = (x - y)^d \cdot (x - y)^d,
\]

the insodual (orthonormal) and the insodual exponential defined respectively by

\[
\log^d x^d = - \log x^d,
\]

\[
\exp^d x^d = \exp x^d \cdot \exp x^d \cdot \exp x^d \cdot \cdots = x^d.
\]

etc. Interested readers can then easily construct the insodual image of special
functions, transforms, distributions, etc.
As an example, an isodual vector field \( \mathbf{X}(x^i) \) on \( E^d \) is given by \( \mathbf{X}(x^i) = -X^i(-x^j) \). The isodual exterior differential of \( \mathbf{X}(x^i) \) is given by

\[
D^d\mathbf{X}(x^i) = \partial_i^d X_j^d x^j + \nabla^d_j x^i x^j = DX(x^i),
\]

where the \( \nabla^d_j \) are the components of the isodual connection. The isodual covariant derivative is then given by

\[
\nabla^d_i \mathbf{X}(x^j) = \partial^d_i X^j x^j + \nabla^d_j x^i x^j - X^j \nabla^d_i x^j.
\]

The interested reader can then easily derive the isoduality of the remaining notions of the conventional geometry.

It is an instructive exercise for the interested reader to work out in detail the proof of the following:

**LEMMA 2.1.6** [8]: The isodual image of a Riemannian space \( \mathcal{R}(x^i, g^d, R^d) \) is characterized by the following maps:

**Basic Unit**

\[
I \rightarrow I^d = -I.
\]

**Metric**

\[
\eta \rightarrow \eta^d = -\eta.
\]

**Connection Coefficients**

\[
\Gamma_{ijk} \rightarrow \Gamma_{ij}^d = -\Gamma_{ijk}.
\]

**Curvature Tensor**

\[
R_{ijk} \rightarrow R_{ij}^d = -R_{ijk}.
\]

**Ricci Tensor**

\[
R^i_{jk} = R^d_i j k = R^i_{jk}.
\]

**Scalar**

\[
R \rightarrow R^d = R.
\]

**Einstein – Hilbert Tensor**

\[
G_{ij} \rightarrow G^d_{ij} = -G_{ij}.
\]

**Electromagnetic Potentials**

\[
A \rightarrow A^d = -A.
\]

**Electromagnetic Field**

\[
B \rightarrow B^d = B.
\]

Figure 2.1. A schematic view of the isodual sphere on isodual Euclidean spaces over isodual fields. The understanding of the content of this chapter requires the knowledge that the isodual sphere and the conventional sphere coincide when imposed by an observer either in the Euclidean or in the isodual Euclidean spaces, due to the identity of the related expressions (2.1.14) and (2.1.15). This identity is at the foundation of the perception that antimatter "appears" to exist in our space, while it is really they belong to a structurally different space coexisting within our one, thus setting the foundations of a "multidimensional universe" consisting in the one-space of our sensory perception. The reader should keep in mind that the isodual sphere is the idealization of the shape of an antiparticle used in this monograph.

2.1.7 Isodual Riemannian Geometry

Consider a Riemannian space \( \mathcal{R}(x^i, g^d, R^d) \) in \( (1 + 3) \) dimensions [32] with basic unit \( I = Diag(1, 1, 1, 1) \), nowhere singular and symmetric metric \( g^d(x) \) and related Riemannian geometry in local formulation (see, e.g., Ref. [37]).

The Riemannian-Santilli isodual spaces (first introduced in Ref. [11]) are given by

\[
\mathcal{R}(x^i, \hat{g}^d(x), \hat{R}^d) \equiv \mathcal{R}(x^i, g^d(x), R^d) = \mathcal{R}(x^i, g^d(x), \hat{R}^d) = \mathcal{R}(x^i, \hat{g}^d(x), R^d),
\]

with interval

\[
x^d = [x^0, x^d(\hat{g}^d(x)); \hat{x}^d = [x^0, x^d(x)]; \hat{x}^d = [x^0, x^d(\hat{x}^d)],
\]

where \( \hat{I} \) stands for transposed.

The Riemannian-Santilli isodual geometries [6] is the geometry of space-time \( E^d \) over \( R^d \), and it is also given by step-by-step isodualities of the conventional geometries, including, most important, the isoduality of the differential and exterior calculus.
for antimatter (see Section 2.3) in such a way that its operator image is indeed the charge conjugate of that of matter.

In this section, we study the physical consistency of the theory in its classical formulation. The novel isodual quantization, the equivalence of isoduality and charge conjugation and related operator issues are studied in the next section.

Beginning our analysis, we note that the isodual theory of antimatter resolves the traditional objections against negative energies and masses. In fact, particles with negative energies and masses measured with negative units are fully spurious in particles with positive energies and masses measured with positive units. This result has permitted the elimination of sole use of second quantisation for the characterization of antiparticles because antimatter becomes treatable at all levels, including second quantization.

The isodual theory of antimatter also resolves the additional, well known, problematic aspects of motion backward in time. In fact, time moving backward measured with a negative unit is fully space-like on grounds of causality to time moving forward measured with a positive unit.

This confirms the plausibility of the first conception of antiparticles by Stueckelberg and others as moving backward in time (see the historical analysis in Ref. [1] of Chapter 1), and creates new possibilities for the ongoing research on the so-called “spacetime machine” studied in Chapter 5.

In this section, we construct the classical isodual theory of antimatter at the Newtonian, Lagrangian, Hamiltonian, Galilean, relativistic and gravitational levels; we prove its axiomatic consistency; and we verify its compatibility with available classical experimental evidence (that dealing with electromagnetic interactions only). Operator formulations and their experimental verifications will be studied in the next section.

2.2.2 Need for Isoduality to Represent All Time Directions

It is popularly believed that time has only two directions, the celebrated Ed- dington’s time arrows. In reality, time has four different directions depending on whether motion is forward or backward and occurs in the future or in the past, as illustrated in Figure 2.2. In turn, the correct use of all four different directions of time is mandatory, for instance, in serious studies of bifurcations, as we shall see.

It is evident that theoretical physics of the 20th century could not explain all four directions of time, since it possessed only one conjugation: time reversal, and this explains the reason the two remaining directions of time were ignored.

It is equally evident that isoduality does indeed permit the representation of the two missing directions of time, thus illustrating its need.

In conclusion, the isodual theory of antimatter correctly represents all available properties of negative charges, a result first achieved in Ref. [9].

2.2.3 Experimental Verification of the Isodual Theory of Antimatter in Classical Physics

The experimental verification of the isodual theory of antimatter at the classical level is provided by the compliance of the theory with the only available experimental data, those on Coulomb interactions.

For that purpose, let us consider the Coulomb interactions under the customary notation that positive (negative) forces express repulsion (attraction) when formulated in conventional Euclidean space.

Under such an assumption, the repulsive Coulomb force among two particles of negative charges $q_1$ and $q_2$ in Euclidean space $E(\vec{r}, \theta, \phi)$ is given by

$$ F = K \times (q_1 \times q_2) \times r \times r \times r \times -F < 0, $$

(2.2.2)

where $|r| = r$ and $\vec{r} = -\vec{r}$ are the usual operations of the underlying field $E(\vec{r}, \theta, \phi)$.

But the isodual force $F = -F$ occurs in the isodual Euclidean space and it is, therefore, defined with respect to the unit $-1$. This implies that the reversal of the sign of a repulsive force measured with a negative unit also describes repulsion. As a result, isoduality correctly represents the repulsive character of the Coulomb force for two antiparticles with positive charges, a result first achieved in Ref. [9].

The formulation of the cases of two particles with positive charges and their antiparticles with negative charges is left to the interested reader.

The Coulomb force between a particle and an antiparticle can only be computed by projecting the antiparticle in the conventional space of the particle or reverse. In the former case we have

$$ F = K \times (q_1 \times q_2) \times r \times r \times r \times 0, $$

(2.2.3)

to yield an attractive force, as experimentally established. In the projection of the particle in the isodual space of the antiparticle, we have

$$ F' = K \times (q_1 \times q_2) \times r \times r \times r \times -F < 0, $$

(2.2.4)

but this force is now measured with the unit $-1$, thus resulting in being again attractive.

The study of Coulomb interactions of magnetic poles and other classical experimental data is left to the interested reader.

In conclusion, the isodual theory of antimatter correctly represents all available classical experimental evidence in the field.

2.2.4 Isodual Newtonian Mechanics

A central objective of this section is to show that the isodual theory of antimatter resolves the scientific imbalance of the 20th century between matter and antimatter, by permitting the study of antimatter at all levels as occurring for matter. Such an objective can only be achieved by first establishing the existence of a Newtonian representation of antimatter subsequently proved to be compatible with known operator formulations.

As it is well known, the Newtonian treatment of $N$ point-like particles is based on a 7N-dimensional representation space given by the Kramer product of the Euclidean spaces of time $t$, coordinates $x$ and velocities $v$ (for the conventional case see Refs. [33],[44],

$$ S(t, x, v) = E(t, R) \times E(x, \dot{R}) \times E(v, \dot{R}), $$

(2.2.5)
throughout the 20th-century it was believed that matter and antimatter exist in the same spacetime. In fact, as recalled earlier, charge conjugation is a map of our physical spacetime into itself.

One of the first physical implications of the Newton-Santilli isodual equations is that antimatter exists in a spacetime coexisting yet different than our own. In fact, the isodual Euclidean space $E^0(2^3, 2^3, 2^3)$ coexists within, but it is physically distinct from our own Euclidean space $E(3, 3, 3)$, and the same occurs for the full representation spaces $B(2^3, 2^3, 2^3)$ and $S(2, 2, 2)$.

The next physical implication of the Newton-Santilli isodual equations is the confirmation that antimatter moves backward in time in a way as caused by the nature of matter forward in time (again, because negative time is measured with a negative unit). In fact, the isodual time $t'$ is necessarily negative whenever $t$ is our ordinary time. Alternatively, we can say that the Newton-Santilli isodual equations provide the only known causal description of particles moving backward in time.

Yet another physical implication is that antimatter is characterized by negative mass, negative energy and negative magnitude of other physical quantities. As we shall see, these properties have the important consequences of eliminating the necessary use of Dirac’s “hole theory.”

The rest of this chapter is dedicated to showing that the above novel features are necessary in order to achieve a consistent representation of antimatter at all levels of study, from Newton to conventional quantization.

As we shall see, the physical implications are truly at the edge of imagination, such as the existence of antimatter in a same spacetime different from our own constitutes the first known evidence of multi-dimensional character of our universe despite our sensory perception to the contrary; the achievement of a fully equivalent treatment of matter and antimatter implies the necessary existence of antigravity for antimatter in the field of matter (and vice-versa); the motion backward in time implies the existence of a causal spacetime machine (although restricted for technical reasons only to isodual states); and other for reaching advances.

2.2.5 Isodual Lagrangian Mechanics

The second level of treatment of matter is that of the conventional classical Lagrangian mechanics. It is, therefore, essential to identify the corresponding formalism for antimatter, a task first studied in Ref. [4] (see also Ref. [9]). A conventional (first-order) Lagrangian $L(t, x)$ on configuration space (2.2.5) is mapped under isoduality into the isodual Lagrangian

$$L^I(t^I, x^I) = -L(t, -x).$$

(2.2.12)

defined on isodual space (2.2.3).

or in the united notation

$$\omega_{\mu} \times \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\mathcal{X}}^\mu} = \frac{\partial \hat{\mathcal{L}}}{\partial \mathcal{X}^\mu} \frac{\partial \hat{\mathcal{X}}^\mu}{\partial \dot{\mathcal{X}}^\mu} \left( \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\mathcal{X}}^\mu} - \frac{\partial \hat{\mathcal{X}}^\mu}{\partial \dot{\mathcal{X}}^\mu} \right) = 0,$$

(2.2.19)

where $\omega_{\mu}$ is the isodual canonical symplectic tensor

$$\left( \omega_{\mu} \right)_{\alpha \beta} = \left( \partial^\mu \hat{\mathcal{X}}^\alpha \partial^\beta \hat{\mathcal{X}}^\beta - \partial^\mu \hat{\mathcal{X}}^\alpha \partial^\beta \hat{\mathcal{X}}^\beta \right) = \left( \omega_{\mu} \right)_{\alpha \beta}.$$  

(2.2.20)

Note that isoduality maps the canonical symplectic tensor into the canonical Lie tensor, with integrating algebraic and geometric implications.

The Hamilton-Jacobi-Santilli isodual equations are then given by [4,9]

$$\frac{\partial \hat{A}^{\mu}}{\partial \dot{\mathcal{X}}^\mu} \frac{\partial \hat{B}^{\mu}}{\partial \dot{\mathcal{X}}^\mu} = 0,$$

(2.2.21a)

$$\frac{\partial \hat{A}^{\mu}}{\partial \dot{\mathcal{X}}^\mu} \frac{\partial \hat{B}^{\mu}}{\partial \dot{\mathcal{X}}^\mu} - \left( \frac{\partial \hat{A}^{\mu}}{\partial \dot{\mathcal{X}}^\mu} \frac{\partial \hat{B}^{\mu}}{\partial \dot{\mathcal{X}}^\mu} \right) = 0,$$

(2.2.21b)

or

$$\omega_{\mu} \times \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\mathcal{X}}^\mu} = \frac{\partial \hat{\mathcal{L}}}{\partial \mathcal{X}^\mu} \frac{\partial \hat{\mathcal{X}}^\mu}{\partial \dot{\mathcal{X}}^\mu} \left( \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\mathcal{X}}^\mu} - \frac{\partial \hat{\mathcal{X}}^\mu}{\partial \dot{\mathcal{X}}^\mu} \right) = -\left[ \hat{A}, \hat{B} \right],$$

(2.2.22)

where

$$\omega_{\mu} = \omega_{\mu} \times \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\mathcal{X}}^\mu} = \frac{\partial \hat{\mathcal{L}}}{\partial \mathcal{X}^\mu} \frac{\partial \hat{\mathcal{X}}^\mu}{\partial \dot{\mathcal{X}}^\mu} \left( \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\mathcal{X}}^\mu} - \frac{\partial \hat{\mathcal{X}}^\mu}{\partial \dot{\mathcal{X}}^\mu} \right) = \omega_{\mu},$$

(2.2.23)

is the isodual symplectic tensor (that coincides with the conventional canonical tensor). The direct representation of isodual equations in first-order form is self-evident. In summary, all properties of the isodual theory at the Newtonian level carry over at the level of isodual Hamiltonian mechanics.

2.2.7 Isodual Galilean Relativity

As it is well known, the Newtonian, Lagrangian and Hamiltonian treatment of matter are only the pre-requisites for the characterization of physical laws via basic relativities and their underlying symmetries. Therefore, no equivalence in the treatment of matter and antimatter can be achieved without identifying the relativities suitable for the classical treatment of antimatter.

To begin this study, we introduce the Galilean symmetry $G(2^1, 1)$ [7,9,22-14] as the step-by-step isodual image of the conventional Galilean symmetry $G(2, 1)$ (hence assumed to be known) [17]. By using conventional symbols for the Galilean symmetry of a Keplerian system of N point particles with non-null masses $m_n$, $n = 1, 2, \ldots, N$, $G^N(2, 1)$ is characterized by isodual parameters and generators

$$\hat{a}(\hat{\mathcal{X}}, \hat{\mathcal{P}}) = \left( \hat{a}^\mu \right)_{\alpha \beta},$$

(2.2.24a)

$$\hat{a}(\hat{\mathcal{X}}, \hat{\mathcal{P}}) = \sum \hat{a}^\mu \hat{\mathcal{X}}^\mu \hat{\mathcal{P}}^\mu = -\hat{\mathcal{X}},$$

(2.2.24b)

$$\hat{a}(\hat{\mathcal{X}}, \hat{\mathcal{P}}) = \sum \hat{a}^\mu \hat{\mathcal{X}}^\mu \hat{\mathcal{P}}^\mu = -\hat{\mathcal{X}},$$

(2.2.24c)

$$\hat{a}(\hat{\mathcal{X}}, \hat{\mathcal{P}}) = \sum \hat{a}^\mu \hat{\mathcal{X}}^\mu \hat{\mathcal{P}}^\mu = -\hat{\mathcal{X}},$$

(2.2.24d)

$$\hat{a}(\hat{\mathcal{X}}, \hat{\mathcal{P}}) = \sum \hat{a}^\mu \hat{\mathcal{X}}^\mu \hat{\mathcal{P}}^\mu = -\hat{\mathcal{X}},$$

(2.2.24e)

equipped with the isodual commutator

$$\left[ \hat{a}(\hat{\mathcal{X}}, \hat{\mathcal{P}}), \hat{a}(\hat{\mathcal{X}'}(\hat{\mathcal{P}}')) \right] = \sum \hat{a}^\mu \hat{\mathcal{X}}^\mu \hat{\mathcal{P}}^\mu = \left( \hat{a} \hat{\mathcal{X}}, \hat{\mathcal{P}}' \right),$$

(2.2.25)

in accordance with rule (2.1.34), the structure constants and Casimir invariants of the isodual algebra $G^N(2, 1)$ are negative-definite. If $\hat{a}(\hat{\mathcal{X}}, \hat{\mathcal{P}})$ is an element of the (connected component) of the Galilei group $G(2, 1)$, its isodual is characterized by

$$\hat{a}(\hat{\mathcal{X}}, \hat{\mathcal{P}}) = \hat{a}(\hat{\mathcal{X}}, \hat{\mathcal{P}}) = -\hat{\mathcal{X}},$$

(2.2.26)

where $R(\theta)\hat{\mathcal{P}}$ is an element of the isodual rotational symmetry first studied in the original proposal [1].

The desired classical nonrelativistic characterisation of antimatter is therefore given by imposing the $G^N(2, 1)$ invariance to the considered isodual equations. This implies, in particular, that the equations admit a representation via isodual Lagrangian and Hamiltonian mechanics.

We now confirm the classical experimental verification of the above isodual representation of antimatter already treated in Section 2.2.2. Consider a conventional, classical, massive particle and its antiparticle in exterior dynamical conditions in vacuum. Suppose that the particle and antiparticle have charge $-e$. 

In this way we reach the basic analytic equations of this chapter, today known as Lagrange-Santilli isodual equations, first introduced in Ref. [6]
**LEMMA 2.2.1 (a)** The trajectories under the same magnetic field of a charged particle in Euclidean space and of the corresponding antiparticle in isodual Euclidean space coincide.

**Proof:** Suppose that the particle has negative charge \(-e\) in Euclidean space \(E(x, y, z)\), i.e., the value \(-e\) is defined with respect to the positive unit \(+1\) of the underlying field of real numbers \(R = R(0, +, \times)\). Suppose that the particle is under the influence of the magnetic field \(B\).

The characterization of the corresponding antiparticle via isoduality implies the reversal of the signs of all physical quantities, thus yielding the charge \(+e\) = \(+s\) in the isodual Euclidean space \(E'(x', y', z', B')\), as well as the reversal of the magnetic field \(B' = -B\), although now defined with respect to the negative unit \((-1)\).

It then evidently that the trajectory of a particle with charge \(-e\) in the field \(B\) defined with respect to the unit \(+1\) in Euclidean space and that for the antiparticle of charge \(+e\) in the field \(-B\) defined with respect to the unit \(-1\) in isodual Euclidean space coincide (Figure 2.3), q.e.d.

An aspect of Lemma 2.2.1, which is particularly important for this monograph, is given by the following:

**COROLLARY 2.2.1 A.** Antiparticles reverse their trajectories when projected from their own isodual space into our own space.

Lemma 2.2.1 assures that isodualities permit the representation of the correct trajectories of antiparticles as physically observed, despite their negative energy, thus providing the foundations for a consistent representation of antiparticles at the level of first quantization studied in the next section. Moreover, Lemma 2.2.1 tells us that the trajectories of antiparticles appear to exist in our space while in reality they belong to an independent space.

### 2.2.8 Isodual Special Relativity

We now introduce isodual special relativity for the classical relativistic treatment of point-like antiparticles (for the conventional case see Ref. [32]).

\(s^2 = x \times t^I\), isodual metric: \(g^I = -\delta\) and basic invariant over \(R^4\)

\((x - y)^I = [(x - y) \times x']^I \in \mathbb{R}^4\) \hspace{2cm} \text{[2.2.29]}

This procedure yields the central symmetry of this chapter indicated in Section 2.2.4, today known as the Poincaré-Santilli isodual symmetry \([7]\).

\[P^I(3,1) = \mathcal{S}(3,1)^I \times \mathcal{T}(3,1)\] \hspace{2cm} \text{[2.2.30]}

where \(\mathcal{S}(3,1)\) is the Lorentz-Santilli isodual symmetry, \(s^2\) is the isodual direct product and \(\mathcal{T}(3,1)\) represents the isodual translations.

The algebra of the connected component \(P_{\text{con}}^I(3,1)\) of \(P^I(3,1)\) can be constructed in terms of the isodual parameters \(s^I = \{-x, -z, +y\} = \{-p, -m, +e\}\) and isodual generators \(X^I = -X = -(-M_{\text{con}} + P_p)\). The isodual commutation rules are given by \([7]\):

\[\left[M_{\text{con}}^I, M_{\text{con}}^J\right] = s^I s^J\left(q_{\text{con}}^{Ia} q_{\text{con}}^{Ja} - q_{\text{con}}^{Ja} q_{\text{con}}^{Ia}\right) - q_{\text{con}}^{Ia} q_{\text{con}}^{Ja} - q_{\text{con}}^{Ja} q_{\text{con}}^{Ia}\] \hspace{2cm} \text{[2.2.31a]}

\[\left[M_{\text{con}}^I, P_p^J\right] = s^I s^J\left(q_{\text{con}}^{Ia} q_{\text{con}}^{Ja} - q_{\text{con}}^{Ja} q_{\text{con}}^{Ia}\right) - q_{\text{con}}^{Ia} q_{\text{con}}^{Ja} - q_{\text{con}}^{Ja} q_{\text{con}}^{Ia}\] \hspace{2cm} \text{[2.2.31b]}

\[\left[P_p^I, P_p^J\right] = 0\] \hspace{2cm} \text{[2.2.31c]}

The Poincaré-Santilli isodual transformations are given by:

\[x'^I = x^I - s^I x^a P_p^a\] \hspace{2cm} \text{[2.2.32a]}

\[x'^I = x^I + s^I x^a P_p^a\] \hspace{2cm} \text{[2.2.32b]}

\[y'^I = y^I - s^I y^a P_p^a\] \hspace{2cm} \text{[2.2.32c]}

\[z'^I = z^I + s^I z^a P_p^a\] \hspace{2cm} \text{[2.2.32d]}

\[t'^I = t^I + s^I t^a P_p^a\] \hspace{2cm} \text{[2.2.32e]}

where

\[s^I = s^I x^a P_p^a - \beta s^I t^a P_p^a = -s^I \beta (1 - \beta^2)^{-1/2}\] \hspace{2cm} \text{[2.2.33]}

and the use of the isodual operations (quotient, square roots, etc.) is assumed.

The isodual symmetrical covering

\[P^{I}(3,1) = \mathcal{S}(3,1)^I \times \mathcal{T}(3,1)\] \hspace{2cm} \text{[2.2.34]}

It should be added that contra to popular belief, the conventional Poincaré symmetry will be shown in Chapter 4 to be three-dimensional, the 0-th dimension being given by a new invariant under change of the unit. Therefore, the isodual symmetry \(P^{I}(3,1)\) is also 3-dimensional.
We finally introduce the isodual electromagnetic waves experience gravitational repulsion when in the field of matter. 

2.2.9 Inequivalence of Isodual and Spacetime Inversions
As it is well known (see, the fundamental spacetime symmetries of the 20-th century are the continuous (connected) component of the Poincare symmetry space reversal (also called parity) and time reversal. As noted earlier, antiparticles are assumed in the above setting to exist in our spacetime and possess symmetries distinct from those of particles, although connected to the latter by the isodual transforms. We shall see, the nontriviality of the isodual special relativity is illustrated by the fact that isodual electromagnetic waves experience gravitational repulsion when in the field of matter. 

2.2.10 Dunning-Davies Isodual Thermodynamics of Antimatter
An important contribution to the isodual theory has been made by J. Dunning-Davies [1] who introduced in 1999 the first, and only known consistent thermo-

\[
\begin{align*}
\text{Figure 2.5.} & \quad \text{A schematic view of the "isodual axis" here defined as a conventional axis with two observers, an external observer in our spacetime and an internal observer in the isodual spacetime. The first implication of the isodual theory is that the same axis exists in the two spacetimes and can, therefore, be detected by both observers. A most intriguing implications of the isodual theory is that such each observer see the other becoming younger. This occurrence is evident for the behavior of the external observer with respect to the external one, since the former evolves according to a time-axiotic that of the latter. The same occurrence is less obvious for the opposite case, the behavior of the internal observer with respect to the external one, and it is due to the fact that the projection of our positive time into the isodual spacetime is indeed a motion backward in that spacetime.}
\end{align*}
\]
and defined on the numbers of antimatter at the operator level.

In fact, the energy-momentum tensor of isodual-electromagnetic waves (2.2.37) is negative-definite [8,9]
\[
T^\mu_\nu = (4 \times 10^{-13} \sigma^p E^p E^\nu + (1/4) \times 10^{-13} \sigma^A E^A E^\nu) \sigma^A F^\mu B^\nu. \quad (2.2.43)
\]
The Einstein-Robert isodual equations for electromagnetism in the vacuum are then given by [8,9]
\[
\sigma^A_B = T^\mu_\nu - \frac{1}{4} \sigma^A \sigma^\nu \sigma^\mu = \frac{4}{10} \sigma^p E^p \sigma^A \frac{1}{4} \sigma^A F^\mu B^\nu. \quad (2.2.44)
\]
The rest of the theory is then given by the use of the isodual Riemannian geometry of Section 2.1.7.

The explicit study of this gravitational theory of antimatter is left to the interested reader due to the indicated inconsistencies of gravitational theories on a Riemannian space for the conventional case of matter (Section 1.2). These inconsistencies multiply when treating antimatter, as we shall see.

2.3 OPERATOR ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.3.1 Basic Assumptions

In this section we study the operator image of the classical isodual theory of the preceding section; we prove that the operator image of antimatter is equivalent to charge conjugation; and we show that isodual mathematics resolves all known objections against negative energies.

A main result of this section is the identification of a simple, structurally new formulation of quantum mechanics known as isodual quantum mechanics, more properly, as the isodual branch of hadronic mechanics first proposed by Santilli in Ref. [9]. Another result of this section is the fact that all numerical predictions of operator isoduality coincide with those obtained via charge conjugation on a Hilbert space, thus providing the experimental verification of the isodual theory of antimatter at the operator level.

Despite this, the isodual image of quantum mechanics is not trivial because of a number of far reaching predictions we shall study in this section and in the next chapters, such as the prediction that antimatter emits a new light distinct from that of matter: antiparticles in the gravitational field of matter experience antimatter. Isodual states of particles and their antiparticles can move backward in time without violating the principle of causality; and other predictions.

2.3.2 Isodual Hilbert Spaces

Isodual quantum mechanics can be constructed via the anti-unitary transform
\[
U \times U^* = H^0 = \mathbb{I} = -1, \quad (2.3.3)
\]
which fixes, for consistency, to the totality of the mathematical and physical formalism of quantum mechanics. We recover in this way the isodual real and complex numbers
\[
\mathbb{R}^d = U \times U^* = \mathbb{C} \times \mathbb{C}. \quad (2.3.4)
\]

The isodual product among generic quantities $A, B$ (numbers, operators, etc.)
\[
A \times B = U (A \times B) U^*, \quad (2.3.5)
\]
and similar properties.

Evidently, isodual quantum mechanics is formulated in the isodual Hilbert space $H^0$ with isodual states [6]
\[
\psi^\dagger \psi^0 = [-1] \psi^\dagger \psi^0 = \psi^\dagger \psi^0, \quad (2.3.7)
\]
where $\psi$ is a conventional dual state on $H$, and isodual inner product
\[
\psi^\dagger \psi^0 = -\psi^\dagger \psi^0 = \psi^\dagger \psi^0, \quad (2.3.8)
\]
with isodual expectation values of an operator $A^0$
\[
A^0 \rightarrow \psi^\dagger \psi^0 A^0 \psi^\dagger \psi^0, \quad (2.3.9)
\]
and isodual normalization
\[
\psi^\dagger \psi^0 = -\psi^\dagger \psi^0 = 1. \quad (2.3.10)
\]
defined on the isodual complex field $C^0$ with unit $-1$ (Section 2.1.1).

The isodual expectation values can also be reached via anti-unitary transform (2.3.3),
\[
\psi^\dagger \psi = \frac{1}{4} \psi^\dagger \psi^0, \quad (2.3.11)
\]

The proof of the following property is trivial.

2.3.3 Isodual Quantization

The isodual Hamiltonian mechanics and its underlying isodual symplectic geometry [5] is not treated in this chapter for brevity; permit the identification of a new quantum channel, known as the isodual quantization [8] that can be readily formulated via the use of the Hamilton-Jacobi-Santilli isodual equations (2.2.21) as follows
\[
\psi^\dagger \psi^0 A^0 \psi^\dagger \psi^0 + \psi^\dagger \psi^0 B^0 \psi^\dagger \psi^0 = 0 \quad (2.3.12)
\]

This property establishes an evident compatibility between the classical and operator formalisms of isodual mechanics. We also mention the isodual energy loss
\[
U^0 (A^0 \times B^0) U^0 = U^0 A^0 B^0 U^0 = \mathbb{I}, \quad (2.3.13)
\]

The isodual true
\[
\psi^\dagger \psi^0 A^0 \psi^\dagger \psi^0 B^0 = \psi^\dagger \psi^0 A^0 B^0 \psi^\dagger \psi^0, \quad (2.3.14)
\]

The isodual determinant
\[
\psi^\dagger \psi^0 A^0 B^0 = \psi^\dagger \psi^0 A^0 B^0 = \psi^\dagger \psi^0 A^0 B^0, \quad (2.3.15)
\]

and other isodual operations.

The interested reader can then work out the remaining properties of the isodual theory of linear operators on a Hilbert space.

2.3.4 Isodualitivy of Minkowski’s Line Elements and Hilbert’s Inner Products

A most fundamental new property of the isodual theory, with implications as vast as the formulation of a basically new cosmology, is expressed by the following lemma whose proof is a trivial application of transform (2.3.3).
LEMMA 2.3.3 [5]: Mimemod’s line elements and Hilbert’s inner products are invariant under isoduality (or they are isoselfdual according to Definition 3.1.2).
\[ \begin{align*}
& x^2 = (\gamma_0 x_0 - x \gamma_0) + i \mathbf{t} \cdot \mathbf{x}^\mathbf{I} \left( \begin{array}{c}
\gamma_0 \\
\mathbf{x}^\mathbf{I}
\end{array} \right), \quad \mathbf{t} = \left( \begin{array}{c}
0 \\
\mathbf{t}
\end{array} \right) = (\mathbf{t}_0, \mathbf{t}). \\
& \gamma_0 = \left( \begin{array}{cc}
1 & 0 \\
0 & -1
\end{array} \right), \quad \mathbf{x}^\mathbf{I} = \left( \begin{array}{c}
x_0 \\
x_I
\end{array} \right), \quad \mathbf{t} = \left( \begin{array}{c}
t_0 \\
t_I
\end{array} \right).
\end{align*} \]
ML. 2.3.3 (c) When, as done on the corresponding isodual spaces defined on isodual Minkowski space $\mathbb{R}^{1,3}$, the above equation is rather consistent with total unit $\mathbb{R}$. The point is that the treatment of antiparticles is no longer restricted to second quantization, as a condition to resolve the scientific imbalance between matter and antimatter indicated earlier.

Condisder the covariant Dirac equation $\square \psi = 0$ with realization of Dirac’s celebrated gamma matrices $\gamma_i = 0, \gamma_0 = 0, \gamma^I = 0, \gamma^{00} = 0 \land \gamma^{I0} = 0$. At the level of first quantization here considered, the above equation is rather universally interpreted as representing an electron under an external electromagnetic field.

For the above equations are generally defined in the 6-dimensional space given by the Kroncker product of the conventional Minkowski space-time and an internal spin space $\mathbb{R}^{(1,3)}(2) \cong SU(2)$. The proof of the following property is recommended to interested readers. THEREM 2.3.3 [5]: Pauli’s sigma matrices and Dirac’s gamma matrices are isodual.

THEOREM 2.3.3 [6]: Dirac’s gamma matrices characterize the direct product of an irreducible two-dimensional (regular) representation of the SU(2) spin symmetry and its isodual, Dirac’s Spin Symmetry: $SU(2) \cong SU(2)$. In fact, the gamma matrices are characterized by the conventional, 2-dimensional Pauli matrices $\sigma_I$ and related identity $I_{0,2}$ as well as other matrices that have resulted in being the exact isodual images $\sigma^I$ with isodual unit $P_{1,3}$. It should be recalled that the isodual theory was born precisely out of these issues and, more particularly, from the incompatibility between the popular interpretation of gamma matrices providing a “four-dimensional” representation of the SU(2) spin symmetry and the lack of existence of such a representation in Lie’s theory.

The sole possibility known to the author for the reconciliation of Lie’s theory for the SU(2) spin symmetry and Dirac’s gamma matrices was to assume that $I_{0,2}$ is the unit of a dual-type representation. The entire theory studied in this chapter then followed. It should also be noted that, as conventionally written, Dirac’s equation is not isodual because it is not sufficiently symmetric in the two-dimensional space and their isoduals.

In summary, Dirac’s work was forced to formulate the “hole theory” for antiparticles because he referred the negative-energy states to the conventional positive unit, while their reformulation with respect to isodual unit $P_{1,3}$ allowed the physical behavior of negative energies in isodual treatment established earlier. Note the complete democracy and equivalence in treatment of the electron and the positron in equation (2.3.28), in the sense that the equation can be equally used to represent an electron or its antiparticle. By comparison, according to the
It should be finally indicated that Abrikosov's treatment of Majorana spinors has a deep connection with isoduality because the underlying Class II spinors have a negative norm [38] precisely as it is the case for isoduality. As a result, the isodual reinterpretation under consideration here is quite natural and actually warranted for mathematical consistency, e.g., to have the topology characterized by a negative norm be compatible with the underlying fields.

2.3.7 Equivalence of Isoduality and Charge conjugation

We come now to another fundamental point of this chapter, the proof that isoduality is equivalent to charge conjugation. This property is crucial for the experimental verification of isoduality at the particle-level too. This equivalence was first identified by Smirnil in Ref. [9] and can be easily expressed today via the following:

**LEMMA 2.3.4 [9,15,18]:** The isodual transform is equivalent to charge conjugation.

**Proof.** Charge conjugation is characterized by the following transform of wavefunctions (see, e.g., Ref. [12], pages 109 and 176):

\[\Psi(x) \rightarrow \Psi^*(x) = -m \times \Phi(x), \tag{2.3.35}\]

where

\[|x| = 1, \tag{2.3.36}\]

thus being manifestly equivalent to the isodual transform

\[\Psi(x) \rightarrow \Phi^*(x) = -\Phi(x). \tag{2.3.37}\]

where \( \Phi \) denotes transpose.

A reason why the two transforms are equivalent, rather than identical, is the fact that charge conjugation maps spacetime into itself, while isoduality maps spacetime into its isodual. Consequently, we have a unitary matrix such that

\[\Phi^* \Phi = \Phi \Phi^* = \mathbf{I}. \tag{2.3.38}\]

is invariant under charge conjugation, in the sense that it is turned into the form

\[c \times (\Phi \Phi^* \Phi) - m \times \Phi = 0, \tag{2.3.39}\]

where the upper four denotes complex conjugation (since \( \Phi \) is a scalar), while the Lagrangian density

\[L = -(\hbar/2)m \times [\Phi^T - i \times \sigma \times A^T(h \times x) + \Phi \times X] \tag{2.3.40}\]

with Lagrangian density

\[L = -(\hbar/2)m \times (\Phi^* + [\Psi^* + (x \times A_{\Phi}(h \times x) \times \Phi)] \times (\Phi^* - \Phi^T); \tag{2.3.41}\]

and four-current

\[J_{\Phi} = i \times \sigma \times A(\Phi \times x) + \Phi \times \gamma_5 \tag{2.3.42}\]

and four-current changes sign, and the four-current remains the same,

\[L_{\Phi} = C_{\Phi} = C_L = C_{\Phi}, \tag{2.3.43}\]

where \( C_L \) is a unitary matrix such that

\[\gamma_5 \rightarrow -\gamma_5 = S_L \times \gamma_5 \times S_L^{-1}. \tag{2.3.44}\]

and there is the change of sign either of \( \Phi \) or of \( \Phi^* \), under which the equation is transformed into the form

\[\delta \Phi^T = \Phi^T \times S_L \times [\Phi^T - \Phi^T] = 0, \tag{2.3.45}\]

while the Lagrangian density changes sign and the four-current remains the same,

\[\lambda = C_L = L = J_{\Phi} + C_{\Phi} = J_{\Phi}, \tag{2.3.46}\]

It is easy to see that isoduality provides equivalent results. In fact, we have for Eq. (2.3.48)

\[\gamma^a \times (\Phi^* + [\Psi^* + (x \times A_{\Phi}(h \times x) \times \Phi)] \times \Phi + m \times \Phi^* = \Phi^* \times \gamma_5 \times \Phi^T \tag{2.3.49}\]

that, when multiplied by \( \gamma_5 \), reproduces Eq. (2.3.13) identically. Similarly, by recalling that Dirac's gamma matrices are isoselfdual (Theorem 2.3.1), and by noting that

\[\Phi^T = (\Phi^* \times \gamma_5)^T = -\gamma_5 \times \Phi, \tag{2.3.50}\]

we have

\[J_{\Phi} = J_{\Phi}, \tag{2.3.51}\]

for the four-current we have

\[\lambda_{\Phi} = -i \times \sigma \times X \times \Phi \times \gamma_5 \tag{2.3.52}\]

But the \( \gamma_5 \) and \( \gamma_5 \) commuting. As a consequence, the four-current does not change sign under isoduality as in the conventional case.

Note that the lack of change of sign under isoduality of Dirac's four-current \( J_{\Phi} \) confirms reinterpretation (2.3.26) since, for the latter equation, the total charge is null.

The equivalence between isoduality and charge conjugation of other equations, such as those by Weyl, Majorana, etc., follows the same line.
whether the selected antiparticle is indeed the isodual image of the particle as a necessary condition for meaningful study of their gravity.

Note that essentially the same ambiguities prohibit the use of mesons for a serious theoretical and experimental studies of the gravity of antiparticles, again, because of unsettled problems pertaining to the structure of the mesons themselves. Since the mesons are naturally unstable, they cannot be credibly believed to be elementary. Therefore, serious theoretical and experimental studies on the gravity of mesons require the prior identification of their constituents with physical particles.

Finally, the reader should be aware that Definition 2.3.2 excludes the use of quark conjectures for the gravitational studies of this monograph. This is due to the well-known basic inconsistency of quark conjectures of not admitting any gravitation at all [see, e.g., the Appendix of Ref. [18]]. In fact, gravity can only be defined in our spacetime while quarks can only be defined in their mathematical unitary internal space with no known connection with our spacetime due to the O'Raifeartaigh theorem.1

Also, the only "masses" that can be credibly claimed as possessing inertia are the pseudo-scalar mass of the second-order Casimir invariant of the Poincare symmetry $p_0 \neq p^0 = m$. Quarks cannot be characterized via such a fundamental symmetry, as well known. It then follows that "quark masses" are mere mathematical parameters defined in the mathematical internal complex-unitary space that cannot possibly be used as serious basis for gravitational tests.

### 2.3.10 Photons and their Isoduals

As it is well known, photons have no charge and, therefore, they are invariant under charge conjugation, as transparent from the simple plane-wave representation

$$
\Phi(x) = \sum_{N \in \mathbb{R}} \psi(x) e^{-iNc \cdot x},
$$

with familiar relativistic form

$$
\Phi(x) = \sum_{N \in \mathbb{R}} \psi(x) e^{-iNc \cdot x},
$$

and familiar expression for the energy

$$
E = h c \nu.
$$

As a result, matter and antimatter have been believed throughout the 20th century to emit the same light. In turn, this belief has left fundamentally unsettled basic quantum questions in astrophysics and cosmology, such as the lack of quantitative

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Note that these particles are not elementary and, as such, they are not admitted by Definition 2.3.1, moreover, as stressed earlier [18], when represented in term of quark conjectures both the proton and the antiproton cannot admit any gravity at all, let alone antimatter. As a result, extreme science care should be exercised before extending to all antimatter any possible gravitational measurements for antiparticles.

### 2.3.13 The Hydrogen Atom and its Isodual

The understanding of this chapter requires the knowledge that studies conducted on the antihydrogen atom (see, e.g., the various contributions in Proceedings [19]), even though evidently interesting per se, have no connection with the isodual hydrogen atom, because the antihydrogen atom has positive mass, for which antimatter is prohibited, and emits conventional photons. Therefore, it is important to inspect the difference between these two formulations of the simplest possible atom of antimatter.

We assume as exact validity the conventional quantum mechanical theory of bound states of point-like particles at large mutual distances, as available in quantum mechanical books as numerous to discourage even a partial listing.

For the case of two particles denoted with the indices 1, 2, the total state in the Hilbert space is the familiar tensorial product of the two states

$$
|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle.
$$

The total Hamiltonian $H$ is the sum of the kinetic terms of such state plus the familiar interaction term $V(|\psi\rangle)$ depending on the mutual distance $r$,

$$
H = p_1^2/2m_1 + p_2^2/2m_2 + g_1 V(|\psi\rangle).
$$

The total angular momentum is computed via the familiar expressions for angular momenta and spins

$$
J = J_1 \otimes I = I \otimes J_2, \quad S = S_1 \otimes I = I \otimes S_2.
$$

where the $I$s are trivial units, with the usual rules for couplings, addition, etc.

One should note that the unit for angular momenta is three-dimensional while that for spin has a generally different dimension.

A typical example of two-body bound states of particles is the hydrogen atom that experiences attraction in the gravitational field of matter with the well established emission of conventional photons.
The study of bound states of point-like isodual particles of large isodual distances is an important part of isodual quantum mechanics. These bound states can be studied via an elementary isoduality of the corresponding bound states for particles, that is, via the use of the isodual Hilbert spaces $H^d$ studied earlier. The total isodual state is the tensoral product of the two isodual spaces

$$|\psi(\mathbf{r})\rangle = |\psi_1(\mathbf{r})\rangle \otimes |\psi_2(\mathbf{r})\rangle,$$  

(2.3.71)

The total isodual spectrum is the sum of the isodual kinetic terms of each particle plus the isodual interaction term depending on the isodual distances,

$$H^d = H_1^d + H_2^d + V(\mathbf{r}).$$  

(2.3.72)

The total isodual angular momentum is expressed in terms of isodual angular momenta and spins

$$J^d = J_1^d + J_2^d + s_1^d + s_2^d,$$  

(2.3.73a)

$$S^d = S_1^d + S_2^d.$$  

(2.3.76)

The remaining aspects (couplings, addition theory of angular momenta, etc.) are then given by a simple isoduality of the conventional theory that is here omitted for brevity.

Note that all eigenvalues that are positive for the conventional case measured with positive units become negative under isoduality, yet measured with negative units, thus achieving full equivalence between particle and antiparticle bound states.

The simplest possible application of the above isodual theory is that for the isodual hydrogen atom (first worked out in Ref. [18]). The novel predictions of isoduality over that of the hydrogen atom is that the isodual hydrogen atom is predicted to experience antigravity in the field of matter and emits isodual photons that are also repelled by the gravitational field of matter.

2.3.14 Isoselfdual Bound States

Some of the most interesting and novel bound states predicted by the isodual theory are the isoselfdual bound states, that is, bound states that coincide with their isodual image. The simplest case is the bound state of one elementary particle and its isodual, such as the positronium.

The condition of isoduality requires that the basic symmetry must be itself isodual, e.g., for the nonrelativistic case the symmetry must be

$$G_{iso} = G(1),$$  

(2.3.74)

antiparticles. Note that the prediction holds only for bound states of truly elementary particles and their isoduals, such as the positronium. No theoretical prediction for the nucleus and the photon is today feasible because the untested nature of their constituents.

2) Isoselfdual bound states are predicted to have a null internal total time $t^d = 0$ and therefore acquire the time of the matter or antimatter in which they are immersed, although the physical time $t$ of the observer (i.e., of the bound state-epiphenomenon) is not null. This is readily understood by noting that the quantity $t$ of Eq. (2.3.77) is our own time, i.e., we merely study the behavior of the state with respect to our own time. A clear understanding illustrated previously with the "isodual rules" of Section 2.1 is that the description of a state with our own time, by no means, implies that its intrinsic time necessarily coincides with our own. Note that a similar situation occurs for the energy because the intrinsic total energy of the positronium is identically null, $E = S^d = 0$. Yet the energy measured by us is $E_{observ} = E_{observ} = 0$. A similar situation occurs for all other physical quantities.

3) Isoselfdual bound states may result in being the microscopic image of the main characteristics of the entire universe. Isoduality has in fact stimulated a new cosmology, the isoselfdual cosmology [21] studied in Chapter 5, that is patterned precisely along the structure of the positronium or of Dirac’s equation in our isodual reinterpretation. In this case the microscopic results in having null total physical characteristics, such as null total energy, null total time, etc., thus implying no discontinuity at its creation.

2.3.15 Resolution of the Inconsistencies of Negative Energies

The treatment of antiparticles with negative energies was rejected by Dirac because of incompatibility with their physical behavior. Despite several attempts made during the 20th century, the inconsistencies either directly or indirectly connected to negative energies have remained unresolved.

The isodual theory of antimatter resolves these inconsistencies for the reason now familiar, namely, that the inconsistencies emerge when one refers negative energies to conventional numbers with positive units, while the same inconsistencies cannot be easily formulated when negative energies are referred to isodual numbers and their negative units.

A good illustration is given by the known objection according to which the creation of a photon from the annihilation of an electron-positron pair, with the electron having a positive energy and the positron a negative energy, would violate the principle of conservation of the energy.

In fact, such a pair could be moved upward in our gravitational field without work and then annihilated in their new upward position. The resulting photon

where $x$ is the Kroncker product (a composition of states thus being isodual), with a simple relativistic extension here assumed from the preceding sections.

The total unit must also be isodual, $E_{tot} = E \otimes E$,  

(2.3.75)

where $E$ represents the space, time and spin units.

The total Hilbert space and related states must also be isodual, $H_{tot} = H \otimes H$,  

(2.3.76a)

and so on.

A main feature is that isoselfdual states exist in both the spacetime of particles and that of antiparticles. Therefore, the computation of the total energy must be done either in $H$ or in $H^d$, in which case the total energy is positive, or in $H_{tot}$, in which case the total energy is negative.

Suppose that a system of one elementary particle and its isodual is studied in our laboratory of matter. In this case the eigenvalues for both particle and its isodual must be computed in $H$, in which case the total energy is negative, as the reader is encouraged to verify.

The total angular momentum and other physical characteristics are computed along similar lines and they also result in having positive values when computed in $H$, as occurring for the conventional charge conjugation.

As we shall see shortly, the positive character of the total energy of bound states of particles and their antiparticles is crucial for the removal of the inconsistencies of theories with negative energy.

The above properties of the isoselfdual bound states have the following implications:

1) Isoselfdual bound states of elementary particles and their isoduals are predicted to be attracted in both, the gravitational field of matter and that of antimatter because their total energy is positive in our world and negative in the isodual world. This renders necessary an experimental verification of the gravitational behavior of isoselfdual bound states, independently from that of individual photons.
Chapter 3

LIE-ISOTOPIC BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

3.1 INTRODUCTION

3.1.1 Conceptual Foundations

As recalled in Chapter 1, the systems generally considered in the 20-th century are the conventional exterior dynamical systems, consisting of closed-isolated and reversible systems of constituents approximated as being point-like while moving in vacuum under sole action-at-a-distance potential interactions, as typically represented by planetary and atomic systems.

More technically, we can say that exterior dynamical systems are characterized by the exact invariance of the Galilean symmetry for the nonrelativistic case and of the Poincare symmetry for relativistic systems. exterior systems are not, since that is, invariant under time reversal. The open-reversible extension of the systems will be studied in the next chapter.

The most important methodological difference between exterior and interior systems is the following:

1) Exterior systems are completely represented with the knowledge of only one quantity, the Hamiltonian, while the representation of interior systems requires the knowledge of the Hamiltonian for the potential forces, plus additional quantities for the representation of nonpotential forces, as done in true Lagrange and Hamilton equations, those with external terms, and, separately, of extended antiparticles, consisting of systems

\[ \frac{d}{dt} \left( \frac{\partial H}{\partial \dot{q}_i} \right) = -\frac{\partial H}{\partial q_i}, \]

\[ \frac{d}{dt} \left( \frac{\partial H}{\partial \dot{p}_i} \right) + \frac{\partial H}{\partial q_i} = F_i(r,v), \]

\[ \frac{d}{dt} \left( \frac{\partial H}{\partial \dot{q}_i} \right) = -\frac{\partial H}{\partial q_i}, \]

\[ \frac{d}{dt} \left( \frac{\partial H}{\partial \dot{p}_i} \right) + \frac{\partial H}{\partial q_i} = F_i(r,v), \]

\[ L = \sum_{i=1}^{N} \left( \frac{1}{2} m_i v_i^2 + V(r,v) \right), \]

\[ H = \sum_{i=1}^{N} \left( \frac{1}{2} \dot{q}_i^2 + \frac{1}{2} \dot{p}_i^2 \right) + V(r,v), \]

\[ F_i(r,v) = F_i(r,v/m). \]

Consequently, by their very conception, interior systems are structurally beyond the representational capability of classical and quantum Hamiltonian mechanics, in favor of covering disciplines.

2) Exterior systems are of Keplerian type, while interior systems are not, since they do not admit a Keplerian center (see, again, Figures 3.1 and 3.2). Consequently, also by their very conception, interior systems cannot be characterized by the Galilean and Poincaré symmetries in favor of covering symmetries.

3) Exterior systems are local-differential, that is, they describe a finite set of isolated points, thus being fully treatable with the mathematics of the 20-th century, beginning with conventional local-differential topologies. By contrast, interior systems are nonlocal-integral, that is, they admit internal interactions over finite surfaces or volumes that cannot be consistently reduced to a finite set of isolated points, respectively. Consequently, interior systems cannot be consistently treated via the mathematics of classical and quantum Hamiltonian mechanics in favor of a basically new mathematics.

4) The time evolution of the Hamiltonian treatment of exterior systems characterizes a canonical transformation at the classical level, and a unitary transformation at the operator level, that we shall write in the unified form

\[ U | \psi \rangle = \psi \quad \text{and} \quad U^\dagger U = I, \]

where $\psi$ represents the usual (associative) multiplication. By contrast, the time evolution of interior systems, being non-Hamiltonian, characterizes nonassociative multiplication.

\[ I \text{ is the identity operator.} \]
studies not fundamentally dependent on the generalization of the basic unit cannot be reviewed for brevity.

3.1.2 Closed Non-Hamiltonian Systems

The first step in the study of hadronic mechanics is the dispelling of the belief that nonpotential forces, being nonconservative, do not permit total conservation laws, namely, that the external terms in the analytic equations (3.1.1) solely apply for open-nonconservative systems, such as an extended object moving within a reactive medium considered as external.

This belief was disproved, apparently for the first time, by Santilli in monographs [1,2]. Ref. [1] presented a comprehensive treatment of the integrability conditions for the existence of a potential or a Hamiltonian, Belinfante’s conditions of variational selfadjointness, according to which the total force is divided into the following two components

\[ F(t, r, v, \ldots) = F^{SA}(t, r, v, \ldots) + F^{NSA}(t, r, v, \ldots), \tag{3.1.4} \]

where the selfadjoint (SA) component \( F^{SA} \) admits a potential and the nonselfadjoint (NSA) component \( F^{NSA} \) does not.

We should recall for clarity that, to be Newtonian as currently understood, a force should solely depend on time \( t \), coordinates \( r \) and velocity \( v = dr/dt \) or moments \( m = mv, F = F(t, r, v) \). Consequentially, forces depending on derivatives of the coordinates of order bigger than the first, such as forces depending on the acceleration \( F = F(t, r, v, a) \), are not generally considered Newtonian forces.

Ref. [2] then presented the broadest possible realization of the conditions of variational selfadjointness via analytic equations derivable from a variational principle, and included the first known identification of closed non-Hamiltonian systems (Ref. [2], pages 233-236), namely, systems that violate the integrability conditions for the existence of a Hamiltonian, yet verify all ten total conservation laws of conventional Hamiltonian systems.

Let us begin by recalling the following well known property:

**THEOREM 3.1.1:** Necessary and sufficient conditions for a system of \( N \) particles to be closed, that is, isolated from the rest of the universe, are that the following ten conservation laws are verified along an actual path

\[ \frac{dX_i(t, r, v)}{dt} = 0, \quad i = 1, \ldots, N. \tag{3.1.5} \]

\[ \dot{X}_i = \dot{E}_i = 0 = H = T + V, \tag{3.1.5a} \]

\[ (X, I, \omega, X_p) = P_m = \Sigma p_m. \tag{3.1.5b} \]

If the nonselfadjoint forces are external.

**DEFINITION 3.1.1 (Ref. 2):** Closed isolated non-Hamiltonian systems of particles are systems of \( N \geq 2 \) particles with potential and nonpotential forces characterized by the following equations of motion

\[ \frac{dX_i}{dt} = \left( \frac{p_{i m}}{m} \right), \tag{3.1.6} \]

varying all conditions (3.1.5), where the term “non-Hamiltonian” denotes the fact that the systems cannot be entirely represented with the Hamiltonian, thus requiring additional quantities, such as the external terms. The case \( n = 1 \) is exceptional, yet admits solutions, and closed non-Hamiltonian systems with \( N = 1 \) obviously cannot exist (because a single free particle is always Hamiltonian).

Closed non-Hamiltonian systems can be classified into:

CLASS a: systems for which Eqs. (3.1.5) are first integrals,

CLASS b: systems for which Eqs. (3.1.5) are consistent relations,

CLASS c: systems for which Eqs. (3.1.5) are subsidiary equations.

The case of closed non-Hamiltonian systems of antiparticles are defined accordingly.

The study of closed non-Hamiltonian systems of Classes b and c is rather complex. For the limited scope of this presentation it is sufficient to see that interior systems of Class a exist.

**THEOREM 3.1.2 (Ref. 2):** Necessary and sufficient conditions for the existence of a closed non-Hamiltonian systems of Class a are that the nonselfadjoint forces

\[ F(t, r, v, a) = \left( \frac{\partial}{\partial a} \right), \tag{3.1.7} \]

forces are external.
verify the following conditions:
\[ \sum \phi_{n}^{(a)} = 0, \quad (3.1.6) \]
\[ \sum p_{n}^{a} + \rho_{n}^{a} = 0, \quad (3.1.7) \]
\[ \sum e_{n} = 0, \quad (3.1.7') \]

**Proof.** Consider the case \( N > 2 \) and assume first for simplicity that \( F^{2} = 0 \). Then, the first nine conservation laws are verified when

\[ \frac{\partial X_{n}}{\partial p_{n}} + F^{2} = 0, \quad (3.1.8) \]

in which the 10th conservation law, Eq. (3.1.5e), is automatically verified, and this proves the necessity of conditions (3.1.7) for \( N > 2 \).

The sufficiency of the conditions is established by the fact that Eqs. (3.1.7) consist of seven conditions on 3N unknown functions \( F_{n}^{2} \). Therefore, a solution always exists for \( N \geq 3 \).

The case \( N = 2 \) is a special instance as motion occurs in a plane, in which case Eq. (3.1.7) reduce to five conditions on four functions \( F_{n}^{2} \), and the system appears to be overdetermined. Nevertheless, solutions always exist because the verification of the first four conditions (3.1.5) automatically implies the verification of the last one, Eqs. (3.1.5e). As shown in Ref. [2], Example 6.3, pages 272–273, a first solution is given by the non-Newtonian force

\[ F_{n}^{2} = -F_{n}^{2} = K \times a = K \times \frac{\partial a}{\partial \xi}, \quad (3.1.9) \]

where \( K \) is a constant. Another solution is given by

\[ F_{n}^{2} = -F_{n}^{2} = M \times \frac{\partial a}{\partial x} \times (M \times x + V), \quad M = \frac{m_{1} + m_{2}}{2}, \quad (3.1.10) \]

Other solutions can be found by the interested reader. The addition of a non-null self-adjoint force leaves the above proof unchanged. q.e.d.

The search for other solutions is recommended to readers interested in acquiring a technical knowledge of hadronic mechanics because such solutions are indeed useful for applications. A general scheme of Eqs. (3.1.7) is well suited as their operator counterpart and of their isodual images for antimatter will be identified later on in this chapter after the identification of the applicable mathematics.

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with more general non-diagonal expressions not considered for simplicity, where \( a_{n}^{k} \), \( a_{n}^{k} \), \( a_{n}^{k} \) represent the semi-axes of the ellipsoidal sphere assumed as a deviation from, or normalized with respect to the perfect sphere

\[ a_{n}^{k} = a_{n}^{k} = a_{n}^{k} = 1. \quad (3.1.12) \]

The \( a_{n} \)’s are called characteristic quantities of the particles considered. It should be stressed that, contrary to a rather popular belief, the \( a_{n} \)-quantities are not parameters because they represent the actual shape as derived from experimental measurements.

To clarify this important point, by definition a “parameter” can assume any value as derived from the list of experimental data, while this is not the case for the characteristic quantities here considered. As an example, the use for the \( a_{n} \)’s of value of the order of \( 10^{-19} \) cm to represent a proton would have no physical value because the electron charge distribution is a spherical ellipsoid of the order of \( 10^{-13} \) cm.

2) Once particles are assumed as being extended, there is the consequential need to represent their density. This task can be achieved via a forth set of quantities

\[ \text{Density of } a_{n}^{k} = \pi a_{n}^{k}, \quad (3.1.13) \]

representing the deviation from the density of the particle considered from the density of the vacuum here assumed to be one,

\[ n_{\text{vacuum}} = 1. \quad (3.1.14) \]

Again, \( a_{n} \) is not a free parameter because its numerical value is fixed by experimental data. As an example for the case of a hadron of mass \( m \) and radius \( r \) we have the density

\[ n_{a} = \frac{4}{3} \pi a^{3} \rho, \quad (3.1.15) \]

thus establishing that \( n_{a} \) is not a free parameter capable of assuming.

Predominantly, most non-relativistic studies can be conducted with the sole use of the three components characterizing the sphere. Relativistic treatments require the additional use of the density as the forth component, resulting in the general form.

\[ \text{Shape - Deviation} = \text{Diag} \left( a_{1}^{k}, a_{2}^{k}, a_{3}^{k}, n_{a}, \right), \quad a = 1, 2, 3, \ldots, N. \quad (3.1.16) \]

3) Perfectly rigid bodies exist in actual abstractions, but not in the physical reality. Therefore, the next need is for a meaningful representation of the deformation of shape as well as variation of density that are possible under interior conditions. This is achieved via the appropriate functional dependence of the characteristic quantities on the energy \( E_{n} \), linear momentums \( p_{n} \), pressure \( P \) and other characteristics, and we shall write

\[ n_{a} = n_{a}(E_{n}, p_{n}, P, \ldots), \quad k = 1, 2, 3, 4, \ldots (3.1.17) \]

The reader is suggested to meditate a moment on the fact that Lagrangian or Hamiltonian systems simply cannot represent the actual shape and density of particles. The impossibility of representing deformations of shapes and variances of density are well-known, since the pillar of conventional non-Hamiltonian structures, the rotational symmetry, is notoriously incompatible with the theory of elasticity.

4) Once particles are represented as they are in the physical reality (extended, non-spherical and deformable), there is the emergence of the following new-class of interactions nonexistent for point-particles (for which reason these interactions have been generally ignored throughout the 20th century), namely, interactions of:

- \( n \)-type, that is, due to the actual physical contact of extended particle; consequently, of
- \( e \)-type, range type, since all contacts are dimensionless; consequently of
- \( m \)-type, point type that is, not representable with any possible action-at-a-distance potential; consequently of
- \( n \)-type, non-Hamiltonian type, that is, not representable with any Hamiltonian; consequently of
- \( v \)-type, conventional type at the classical level and non-conventional type at the operator level; as well as of
- \( w \)-type, nonlinear type, that is, represented via nonlinear differential equations, such as being supported on power of the wavefunction greater than one; and, finally, of
- \( n \)-type, nonlinear integral type. Interactions among point-particles are local-differential, that is, reducible to a finite set of isolated points, while contact interactions among extended particles and/or their wavepackets are, by conception, nonlocal-integral in the sense of being dependent on a finite surface or volume that, as such, cannot be reduced to a finite set of isolated points (see Figure 3.3).

5) Once the above new features of interior systems have been identified, there is the need not only of their mathematical representation, but above all of their corresponding representation in order to abstract the scheme of catastrophe inconsistencies of Chapter 1.

As an illustration, Coulomb interactions have reached their towering position in the physics of the 20th century because the Coulomb potential is invariant under the basic symmetries of physics, thus predicting the same numerical values under the same conditions at different times with consequently consistent physical applications. The same occurs for other interactions derivable from a potential (except gravitation represented with curvature as shown in Section 1.4).

Along the same lines, any representation of the extended, non-spherical and deformable character of particles, their densities and their novel nonlinear, non-
local and nonpotential interactions cannot possibly have physical value unless it is also recurrent, and not "covariant," again, because the latter would violate the theorems of catastrophic inconsistencies of Chapter 1. It should be indicated that an extensive search conducted by the author in 1978-1983 in the advanced libraries of Cambridge, Massachusetts, identified numerous integral geometries and other nonlocal mathematics. However, none of them verifies all the following conditions necessary for physical consistency:

CONDITION 1: The new nonlocal-integral mathematics must admit the conventional unit of multiparticle and nondiagonal forms, where the \( s_k \) represents the numbers of the isoscalar shape of particle \( k \). The expression \( I' \) represents the density, the expression \( I'(r, \psi(r, \psi) \cdots) \) represents the nonlocality of the interaction and \( I' \) is local, \( \psi'(r, \cdots) \) provides a simple representation of its nonlocality. The corresponding features of antiparticles are represented by Santilli's isounit \( I' \), and mixed states of particles and antiparticles are represented by the tensorial product of the corresponding units and their products.

In this chapter we shall merely present recent developments in the construction of isomathematics that have occurred following the publication of the second edition of Vol. I of this series in 1995 [8] since these developments have important implications. We shall then identify the recent developments in physical theories occurred since the second edition of Vol. II of this series [9]. We shall then review the novel industrial applications developed since the appearance of Volumes I and II. It should be noted that in this chapter we shall merely review recent developments. As a consequence, Volumes I and II of this series [8] remain useful for all detailed aspects that will not be repeated in this final volume. A primary motivation of this volume is to present industrial applications. Consequently, we have selected the simplest possible mathematical treatment accessible to any experimentalists. Readers interested in utmost mathematical rigour are suggested to consult the specialized mathematical literature in the field.

Finally, Santilli isounit \( I \) identifies in full the covering nature of isomathematics over conventional mechanics, as well as the fact of reducing the conventional to a lower level of effective isounits for the whole of high-energy interactions. This covering character is illustrated by the fact that at sufficiently large mutual distances of particles the integral in the exponent of Eq. (3.1.19) is null.
3.2 ELEMENTS OF SANTILLI’S ISOMATHEMATICS AND ITS ISODUAL

3.2.1 Isounits, Isoproducts and their Isoduals

As indicated earlier, Santilli isometric number theory, [4-8] or isomathematics for short, is characterized by the map, called lifting, of the trivial unit \(I = e^1\) into a generalized unit \(I\) of the isomathematics. Thus, for \(m\)-dimensional units

\[
I = e^1 \to (t, r, p, \psi, \beta, \phi, \psi', \beta', \phi') = \mathcal{D}(1, 1, \ldots, 1, i, j, k, \ldots, N),
\]

(3.2.1)

of conventional Hamiltonians into a nonassociative, nonunitary, and noncanonical unitary system \(\mathcal{D}(1, 1, \ldots, 1, i, j, k, \ldots, N)\) of \(m\)-dimensional units \(A\) and \(B\) over \(m\)-dimensional \(m\)-space are based on the unit \(I\) of Eq. (3.2.1), that is, the basic isounit of the theory of hadronic mechanics.

The reader should be aware that other generalizations of the associative product, such as the isodifferential calculus that escapes construction via noncanonical-nonunitary transforms.

For example, once the fundamental realization (3.2.9) is assumed, the construction of isomathematics is indeed recommended for physicists as a generalization of the theory of the Gel’fand-Fomin’s formalism.

Isomathematics is the study of isounits, isoproducts, and their isoduals, i.e., the study of the lifting of \(N\)-dimensional Hamiltonian systems into a nonassociative, nonunitary, and noncanonical system, such as the isodifferential calculus that escapes construction via noncanonical-nonunitary transforms.

The basic quantity of isodual isomathematics is then the isounit, whose elements \(\mathcal{D}(1, 1, \ldots, 1, i, j, k, \ldots, N)\) are guaranteed because of the property,

\[
\mathcal{D}(1, 1, \ldots, 1, i, j, k, \ldots, N) = \mathcal{D}(1, 1, \ldots, 1, i, j, k, \ldots, N),
\]

(3.2.10)

Once the fundamental realization (3.2.9) is assumed, the construction of isomathematics follows in a simple, unique and unambiguous way. In fact, isomathematics can be constructed by submitting conventional mathematics with left and right unit \(I\) to said noncanonical-nonunitary transforms, with very few exceptions, such as the indeterminable calculus that escapes construction via noncanonical-nonunitary transforms.

At the classical level, here written in the unified form

\[
\mathcal{D}(1, 1, \ldots, 1, i, j, k, \ldots, N),
\]

(3.2.11)

for all elements \(A\) of the considered set. In this case (only) \(I\) is called Santilli’s isometric unit, or isounit for short, and \(I\) is called Santilli’s isometric element, or isounit for short.

Isomathematics was first submitted by Santilli in monographs [loc. cit.] in 1976 and then worked out in various additional contributions by the same author, as well as by numerous mathematicians and theoretical physicists (see the references of Chapter 1 as well as of this section).

The most salient feature of Santilli’s liftings (3.2.2) and (3.2.3) is that they are not isotopic, from which feature may derive their name "isotopic" [loc. cit.], correctly contrasted to the prefix "iso-

In fact, \(I\) preserves the basic topological characteristics of \(I\). Therefore, isomathematics is expected to provide new realizations of the abstract axioms of the mathematics admitting \(I\) as left and right unit. In particular, the preservation of the original abstract axioms is an important guiding principle in the consistent construction of isomathematics and their applications.

At this introductory stage the axiom-preserving character of generalized product (3.2.3) is easily verified by the fact that it preserves all basic axioms of the original product. In fact, the isoproduct verifies the right and left inner law

\[
A(B \cdot C) = (A \cdot B) \cdot C,
\]

(3.2.1a)

(3.2.1b)

and the right and left distributive laws

\[
A(B + C) = A \cdot B + A \cdot C,
\]

(3.2.2a)

(3.2.2b)

and the associative law

\[
A \cdot (B \cdot C) = (A \cdot B) \cdot C,
\]

(3.2.3a)

A verification of the preservation of the axioms of all subsequent constructions is crucial for a serious study and application of hadronic mechanics. The simplest construction of isomathematics as needed for various applications is given by the use of a positive-definite \(N\)-dimensional noncanonical transforms at the classical level or a noncanonical transforms at the oper-

ator level, here written in the unified form

\[
U \cdot U = I \neq 0
\]

(3.2.4a)

The reader should be aware that other generalizations of the associative product, such as

\[
A 
\]

(3.2.14a)

(3.2.14b)

are unacceptable because they violate either the right or the left distributive and scalar laws, thus being unable to characterize an algebra. As such, liftings (3.2.14) are not isotopic in Santilli’s sense [loc. cit.]

Examples of isomathematics have been given in Section 3.1.3. Additional examples will be provided in Sections 3.3 and 3.4. Note that, since they are Hamiltonian by assumption, isounits can always be diagonalized into the form of type (3.1.19).

3.2.2 Isounit and the same simple transform holds for the construction of other aspects of isomathematics, as illustrated in this section.

As a matter of fact, the use of the above transform provides a method for the construction of isomathematics that is more rigorous than empirical liftings. For instance, by comparing Eqs. (3.2.3) and (3.2.13), we see that the lifting of the unit \(I = I = U \times I \times U^0\) implies not only the lifting of the associative product

\[
A \cdot B = U \times (A \times B) = (U \times A) \times U^0 + (U \times B) \times U^0 = U \times (A \times B) = U \times (A \times B),
\]

(3.2.13)

and the same simple transform holds for the construction of other aspects of isomathematics, as illustrated in this section.

In view of the above, the claim often expressed in the nonmathematical physics literature that “the mathematics of hadronic mechanics is too difficult to comprehend” is just a case of venturing judgment without any serious knowledge of the topic.

I = U \times U = I \neq I
\]

(3.2.12)

The reader should be aware that other generalizations of the associative product, such as

\[
A 
\]

(3.2.1a)

(3.2.1b)

are unacceptable because they violate either the right or the left distributive and scalar laws, thus being unable to characterize an algebra. As such, liftings (3.2.14) are not isotopic in Santilli’s sense [loc. cit.]

Examples of isomathematics have been given in Section 3.1.3. Additional examples will be provided in Sections 3.3 and 3.4. Note that, since they are Hamiltonian by assumption, isounits can always be diagonalized into the form of type (3.1.19).
In fact, the use of anti-isomorphic transforms causes ambiguities in the very central issue, the achievement of equivalence of the medial operator theory with charge conjugation due to ambiguities and other technical aspects. In turn, this occurrence illustrates the significance and uniqueness of Santilli isodual map (1.2.15).

Note also that isodual isomathematics preserves the axioms, not of conventional mathematics, but of the isodual mathematics of Chapter 2, that with the simplest possible isounit map $I^+ = I$

Needless to say, mathematicians do not need the above elementary construction of isomathematics and its isodual since they can be formulated on abstract realization-free grounds from basic axioms.

### 3.2.2 Isonumbers, Isofields and their Isoduals

The first necessary isotope lifting following that of the basic unit and product, is that of ordinary numbers. The resulting new numbers were first presented by Santilli at the 1960 meeting in Choralnd, Germany, on "Differential Geometry Methods in Mathematical Physics" and then published in a variety of papers, such as Ref. [6] of 1965, Vol. 15(8) of 1991, memoir [9] of 1993 and other works. A comprehensive presentation is available in Vol. I of 1995 that also presents industrial applications of the new numbers for cryptograms and other fields. As a result of these contributions the new numbers are today known as Santilli's isonumbers.

The new numbers have also been studied by various authors. An important contribution has been made by E. Toll [1] in 1998 consisting in a proof of Fermat's celebrated theorem that is the simplest on record and, therefore, credibly conceivable by Fermat (as opposed to other proof requiring mathematics basically unknown during Fermat's time). Unfortunately, Fermat left no record of the proof of his celebrated theorem and, therefore, there is no evidence that Fermat first studied numbers with arbitrary units. Nevertheless, Toll's proof of Fermat's theorem remains the most plausible known to this author for being conceived during Fermat's time.

Numerous additional studies on isonumbers have been conducted by other authors. For a complete bibliography we refer interested readers to the monograph on Santilli isonumber theory by C.-X. Jiang [12] of 2002. Additional studies on isonumbers have occurred for the use as basis of other isonumbers.

Related references will be quoted in the appropriate subsequent sections.

Santilli's isonumbers have also been subjected to a generalization called pseudo-isonumbers identified in Ref. [6] and studies by various authors, including N. Kamiya [3] and others. However, the latter generalization violates the axioms of a field and, as such, it cannot be used for hadronic mechanics.

For the use of a positive-definite (thus invertible) noncanonical-nonunitary isounit, with clearly added safety.

An important particular case is the property that

$$a^2 = a \times \cdots \times a \quad (n \text{ times}) = a^n \times I.$$  

(3.2.20)

An important particular case is the property that the isounits of the isonumber reproduce the isounit identically:

$$F = I \times I \times \cdots \times I = I.$$  

(3.2.21)

Similarly we have the isounit isonumbers

$$a^{1/2} \equiv a^{1/2}, \quad I^{1/2},$$

(3.2.22)

the isounit

$$a^{1/2} = (a^2) \times I \equiv (a^n) \times I,$$

(3.2.23)

and the isounit

$$| \equiv | a | I.$$  

(3.2.24)

where $| a |$ is the conventional norm. All these properties were first introduced by Santilli in Refs. [8–9]. The reader can now easily construct the desired isounit image of any operation on numbers. Despite their simplicity, isonumbers are nontrivial. As an illustration, the assumption of the isounit $I = 3$ implies that "2 multiplied by 3" = 18, while 4 becomes a prime number.

The best way to illustrate the meaningfulness of the new numbers is to indicate the industrial applications of Santilli's isonumbers, that are a primary objective of this monograph as indicated earlier.

To begin, all applications of hadronic mechanics are based on isonumbers, and they will be presented later on in this chapter. In addition to that, Santilli's isonumbers have already found a direct industrial applications consisting of the isotope lifting of cryptograms said by the industry to protect secrecy, including banks, credit cards, etc. This industrial application was first presented by Santilli in Appendix B.C. of the second edition of Vol. I of 1995 and will be reviewed later on in this chapter.

At this moment we merely mention that all cryptograms based on the multiplication depend only on one value of the unit, the quantity $1$ dating back to biblical times. A mathematical theorem establishes that a solution of any cryptogram can be identified in a finite period of time. As a result of this occurrence, banks and other industries are forced to change continuously their cryptograms to properly protect their secrecy.

By comparison, Santilli's isocryptograms are based on the isoprodact and, as such, they admit an infinite number of possible isounits, such as, for instance, the values

$$I = 7.2, \quad 9.9364, \quad 23.6, \quad 1.295 \times 10^4, \quad 6.$$  

(3.2.24)

Consequently, it remains to be seen whether Santilli can isocryptograms can be broken in a finite period of time under the availability of an infinite number of possible isounits.

Independently from that, with the use of isocryptograms banks and other industries do not have to change the entire cryptogram for security, but can merely change the value of the isounits to keep ahead of possible hackers, and even that process can be computerized for frequent automatic changes of the isounit, with clearly added safety.

Finally, another application of Santilli isocryptograms permitted by their simplicity is their use to protect the access to personal computers. It is hoped this illustrates the industrial significance of Santilli isonumbers per se, that is, independently from their basic character for hadronic mechanics.

We now pass to a mathematical presentation of the new numbers.

**Definition 3.2.1 [10].** Let $F = F(a, b, c)$ be a field of characteristic zero as per Definition 2.1.1. Santilli's isofields are rings $F = F(a, b, c)$ with elements

$$\hat{a} = a \times I,$$

(3.2.25)

where $a \in F$, $I = 1/I$ is a positive-definite quantity generally outside $F$ and $x$ is the ordinary product of $F$; the isounit is an isounit with the ordinary sum $+$, $\hat{a} + \hat{b} = \hat{a} + \hat{b}, \quad a \in F$ (3.2.33)

and the isounit $\times$ such that $I$ is the right and left isounit of $F$,

$$I \times I = I \times \hat{a} = \hat{a}, \quad a \in F.$$  

(3.2.34)

Santilli's isofields verify the following properties:

1. For each element $\hat{a} \in F$ there is an element $\hat{a}^{-1}$, called isounverse, for which

$$\hat{a} \times \hat{a}^{-1} = I, \quad \hat{a} \in F.$$  

(3.2.35)

2. The isounit is isounverse

$$\hat{a} \times \hat{b} = \hat{b} \times \hat{a},$$

(3.2.36)

and isounassive

$$(\hat{a} \times \hat{b}) \times \hat{c} = \hat{a} \times (\hat{b} \times \hat{c}), \quad \forall \hat{a}, \hat{b}, \hat{c} \in F.$$  

(3.2.37)
...the axioms of a field require that the multiplicative unit to be the identity of a field).

The above liftings result in:

\[ R^2(1); 1, 2, \ldots, N; \]

and the isodual isoproduct

\[ \text{Isodual isoproduct: } (\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = (\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c}; \]

and the isodual isocomplex isonumber

\[ (\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = (\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c}; \]

and the basic character of the unit should be recalled here. For the case of the three-dimensional Euclidean space, the basic unit is not only the basic geometric unit, but also the unit of the entire Lie theory and Hamiltonian mechanics.

**LEMMA 3.2.3 [9]:** Isodual isocomplex isonumbers are anti-isomorphic to isonumbers.

As we shall see in this chapter, the latter property, jointly with the anti-isomorphic character of the isonumbers, will result to be crucial for a consistent treatment of antimatter composed of extended particles with potential and non-potential internal forces.

The above properties establish the fact (first identified in Ref. [8]) that, by no means, the axioms of a field require that the multiplicative unit to be the trivial unit \( +1 \), because the basic unit can be a negative-definite quantity \(-1\) as it occurs for the isodual mathematics of Chapter 2, an arbitrary positive-definite quantity \( T > 0 \) as occurring in isometrics, or an arbitrary negative-definite quantity \( T < 0 \) as it occurs for the isodual isomathematics.

The reader should be aware that in depth knowledge of Santilli's isonumbers and their isocomplex requires an in-depth study of monograph [9] or of Chapter 2 of Vol. I of this series, Ref. [8], and that an in-depth knowledge of Santilli's isoomathematics requires a study of Jiang's monograph [12].

Finally, the reader should meditate a moment on the viewpoint expressed several times in this writing to the effect that there cannot be truly new physical theories without new mathematics, and that the basic novelty of hadronic mechanics can, therefore, be reduced to the novelty of Santilli's isonumbers.

By remembering that all "numbers" have been fully identified constructively as the basic novelty of hadronic mechanics can be reduced to the discovery that the axioms of conventional fields admit new realizations with nonsingular, but otherwise arbitrary multiplicative units.

---

**3.2.3 Isonumbers and Their Isoduals**

Following the lifting of units, products and fields, the next necessary lifting is that of N-dimensional metric or pseudo-metric spaces with local coordinates \( r \) and Hamiltonian, thus diagonalized metric in over a field \( F \), here written in the unified notation

\[ S(r, m, F) = I(r) = m_{ij}(\mathbf{\epsilon} \cdot \mathbf{r}) = \text{Diag}(m_{11}, m_{22}, \ldots, m_{NN}); \]

\[ r, j, k = 1, 2, \ldots, N; \]

basic invariant

\[ r^2 = (r \times m_{ij} r) \times I = (r \times m \cdot r) \times I = F(u, +, \chi); \]

(3.56)

where \( I \) stands for transpose and fundamental N-dimensional unit

\[ I = \text{Diag}(1, 1, \ldots, 1) = I = (r \cdot r) = U \times U^\dagger = 1/2 \quad \text{Tr} \theta = 3. (3.56) \]

The above liftings requires that of spaces \( S(r, m, R) \) \( = \) isometric spaces, or isospaces for short, for the treatment of matter, hereon denoted \( S_{/\mathcal{M}, F} \), where \( \mathcal{M} \) denotes the isomatrix, or \( \mathcal{M} \), and \( F \) \( = \) \( F(\mathbf{a}, +, \chi) ; \) of Section 3.2.2

Isonumbers were first proposed by Santilli in Ref. [14] of 1983 for the anti-processing isometrics of the Kharkovian spacetime and special relativity that are at the foundations of hadronic mechanics. Isonumbers were then used by Santilli for the lifting of the various spacetime and internal symmetries (such as \( SU(2) \), \( SU(3) \), \( SU(1, 1), \) \( \mathcal{E}(C) \), \( G(2, 1), \) \( F(3, 3), \) \( \mathcal{S}(3, 3) \), etc.) as studied later on in this chapter.

Following the appearance of these contributions, isospaces have been also studied by a number of authors for both mathematical and physical applications to be studied in subsequent sections, including the definition of incontinuity, isometry, isosstructures, etc. An introduction will be presented in the appropriate subsequent sections.

In this section we identify the basic notions of Santilli isospaces. Specific types of isospaces needed for applications will be studied in subsequent sections. The coordinates $r$ of ordinary spaces $S(r, m, F)$ are defined on the base field $F = \mathbb{R}(a_1, \ldots, s_1)$ by the system of $F$-complex numbers for $F = R$ (conventional numbers for $F = C$) and quaternionic numbers for $F = Q$.

Consequently, the isocomponents $r$ on isospaces $S(r, m, F)$ must be defined on the isofields $F = \mathbb{R}(a_1, \ldots, s_1)$, since the multiplication of the invariant by the unit is trivial for conventional studies and, as such, it was ignored.

As we shall see in the next section, the above definition is independently confirmed by construction.

Moreover, deformed spaces $S(r, m, F)$ necessarily break the symmetries of the original spaces $S(r, m, F)$, while, as we shall soon see, isospaces $S(r, M, F)$ reconstruct the exact symmetries of $S(r, m, F)$.

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Thus, Santilli's isospaces are the $N$-dimensional covector spaces

$$S(r, M, F) : r = (r) \times I \in F$$

$$M = (I, x) \times \mathbb{R}(a_1, \ldots, s_1) \in F,$$

$$\phi^2 = (n_0, c_0, s_0) \times s, I = n_0 M \times d = 0 = m \times x, x \times l \times F \neq Q.$$
isofunctions. For self-sufficiency of this volume we recall that their definition requires isofunctions. Ref. [7]. For self-sufficiency of this volume we recall that their definition requires

The lifting of trigonometric functions is intriguing and instructive (see Chap. 3). For the case of the simple function

Let $M(x)$ be an $N$-dimensional matrix with elements $M_{ij}(x)$ on a conventional space $S(x,F)$ with local coordinates over a conventional field $F$ with unit $I$. Then, the isotropic image of $M(x)$ or its isomatrix, is defined by $\tilde{M}(x) = (M_{ij}(x)) = M(U \times x) \times \hat{I}$, $M_{ij} \in F$. (3.2.74)

Similarly, the isodeterminant of $\tilde{M}$ is defined by

\[ \text{Det}[\tilde{M}] = \text{Det}([\tilde{T} \times \tilde{M}]) \times \hat{I}, \] (3.2.75)

where Det represents the conventional determinant, with the preservation of the conventional axioms, e.g.,

\[ \text{Det}(\tilde{M}_1 + \tilde{M}_2) = \text{Det}(\tilde{M}_1) + \text{Det}(\tilde{M}_2); \] (3.2.76a)
\[ \text{Det}(\tilde{M}^{-1}) = (\text{Det}[\tilde{M}])^{-1}. \] (3.2.76b)

Note that, by construction, isomatrices and isodeterminants preserve the original values on isospaces over isodetachable, although slow deviations when the same quantities are observed from the original space, that is, referred to the original unit $I$.

Similarly, the isosine of $\tilde{M}$ is defined by

\[ \tilde{\text{Sin}}[\tilde{M}] = [\tilde{T} \times \hat{I}] \times I, \] (3.2.77)

as one can see, expression (3.2.68) coincides with the definition of isofunction in the quoted references. A feature identified since that time is the re-interpretation in such a way that the function $f(x)$ preserves its numerical value when formulated as $f(x)$ on the isospace $S$ over the isofield $F$ because the variable $x$ is multiplied by $T$ while the unit to which such a variable is referred to is multiplied by the inverse amount $I = 1/T$. All numerical differences emerge in the projection of $f(x)$ in the original space. This is essentially the definition of isofunctions that will allow us to preserve the basic axioms of special relativity on isospaces over isodetachable and actually expand their applicability from motion in empty space to motion within physical media.

For the case of the simple function $f(x) = x$ we have the lifting

\[ \tilde{x} = U \times x \times U^T = x \times (U \times U^T) = x \times I = x \times \hat{I} \in F, \] (3.2.71)

with the projection in the original space $S$ being simply given in this case by $T \times x$.

More instructive is the lifting of the exponention into the isonexponention given by

\[ \hat{e}^x = U \times e^x \times U^T, \] (3.2.70)

where $Tr$ is the conventional trace, and it also verifies the conventional axioms, such as

\[ \text{Tr}(\tilde{M}_1 \times \tilde{M}_2) = \text{Tr}(\tilde{M}_1) \times \text{Tr}(\tilde{M}_2), \] (3.2.78a)
\[ \text{Tr}(\tilde{M}^{-1}) = \text{Tr}(\tilde{M})^{-1}. \] (3.2.78b)

The isotrace is hence defined by

\[ \hat{\text{Log}}[\hat{\text{Log}}] = \text{Log}(\text{Log}) \times \hat{I}, \] (3.2.80)

under which the conventional axioms are preserved,

\[ \hat{\text{Log}}[\hat{\text{Log}}] = \hat{I}, \] (3.2.81a)
\[ \hat{\text{Log}}[\hat{I}] = 0, \] (3.2.81b)
\[ \hat{\text{Log}}(x \times y) = \hat{\text{Log}}(x) + \hat{\text{Log}}(y), \] (3.2.81c)
\[ \hat{\text{Log}}(a^{-1}) = -\hat{\text{Log}}(a), \] (3.2.81d)
\[ \hat{\text{Log}}(x^a) = \hat{\text{Log}}(a x), \] (3.2.81e)

The lifting of trigonometric functions is intriguing and instructive (see Chapter 6 of Ref. [6] and Chapter 5 of Ref. [7]) whose results in this case require no upgrade. Let $E(r, \hat{R})$ be a conventional two-dimensional Euclidean space with coordinates $r = (x, y)$ on the reals $R$ and polar representation $x = r \times \cos \theta$ and $y = r \times \sin \theta$, $x^2 + y^2 = r^2 \times (\cos^2 \theta + \sin^2 \theta) = r^2$. Consider now the iso-Euclidean space in two-dimensions.

\[ \tilde{E}(\hat{r}, \hat{R}) : \hat{r} = \text{Diag}(y, x^2), \hat{R} = \text{Diag}(x^2, y^2). \] (3.2.82a)
\[ \hat{r}^2 = (\hat{x}^2 + \hat{y}^2)^2 \times I \in R. \] (3.2.82b)

Then, the isoinvariant coordinates and related isotrigonometric functions on $\hat{R}$ are defined by

\[ \hat{\theta} = r \times \sin \theta, \] (3.2.83a)
\[ \hat{\cos} \theta = r \times \cos \theta \times (x/n_1 \times y/n_2), \] (3.2.83b)

where $n_1 \times n_2$ is the vectorial product of $n_1$ and $n_2$.

Note, again, that a different definition of isoinvariances was announced in Eq. (3.2.75) of Ref. [6].

\[ \hat{r}^2 = r^2 \times (n_1 \times n_2)^2 \times I \in R. \] (3.2.84)

The isotopy of spherical coordinates are treated in detail in Section 5.5 of Ref. [7]. For self-sufficiency of this volume we recall that their definition requires a three-dimensional iso-Euclidean space $E(\hat{r}, \hat{R}) : \hat{r} = \text{Diag}(y, x^2, n_1^2, n_2^2), \hat{R} = \text{Diag}(x^2, y^2, n_1^2, n_2^2).$ The isotopy of the conventional spherical coordinates $E(r, \hat{R})$ then yields the following isospherical coordinates here presented in the projected form in $E(\hat{r}, \hat{R})$.

\[ x = r \times n_1 \times \sin \theta(n_1 \times n_2) \times \sin \phi(n_1 \times n_2), \] (3.2.86a)
\[ y = r \times n_2 \times \sin \theta(n_1 \times n_2) \times \cos \phi(n_1 \times n_2), \] (3.2.86b)
\[ z = r \times n_1 \times \cos \theta(n_1 \times n_2). \] (3.2.86c)

Via the use of the above general rules, the reader can now construct all needed isoditions.
then passing to the correct invariant formulation of all dynamical equations of a mathematical presentation in memoir [10] of 1996. The resulting generalization hadronic mechanics.

The above problem was finally resolved by Santilli in the second edition of his book on isodifferential calculus, the impasse.

The reader should be aware that in most applications of hadronic mechanics the isounits can be interchanged, resulting in possible ambiguities that could cause loss of invariance for applications and verifications of hadronic mechanics. This update is recommended because of various presentations in which the role of \( I \) and \( T \) were interchanged, resulting in possible ambiguities that could cause loss of invariance even under the lifting of the differential calculus.

A main feature is that, unlike all other aspects of hadronic mechanics, the isospace over iso-fields are curves. Therefore in isospace we have expressions like

\[
\frac{\text{d}x^i}{\text{d}t} = (2/3)^{1/2} \int \text{d}t' (\frac{1}{2})^{1/2} \frac{\partial x^i}{\partial u^j} \text{d}u^j.
\]

This is due to the fact that conventional triangles and the Pythagorean theorem are preserved in isospace, they provide a generalization of the Pythagorean theorem to curvilinear triangles, and this explains the delay in the resolution of this problem.

In this section we review Santilli’s isodifferential calculus in its version needed for applications and verifications of hadronic mechanics since its proposal in 1978 [5] was due to difficulties in identifying the origin of the non-invariance of its initial formulation, that is, the lack of prediction of the same numerical values for the same quantities under the same conditions, but at different times, a fundamental invariance property fully verified by quantum mechanics.

These difficulties were related to the lack of a consistent isometric lifting of the familiar quantum mechanical momentum. More particular, all aspects of quantum mechanics could be consistently and easily lifted via a non-differential transforms, except the eigenvalue equation for the linear momentum, as shown by the following lifting

\[
p \times \phi (x) \rightarrow \rho \times \phi (x) = \mathbf{K} \times \phi (x) \rightarrow \mathbf{K} \times \phi (x) = \rho \times \phi (x) = \mathbf{p} \times \phi (x).
\]

Let \( E(\mathbf{r}, M, R) \) be an isotope of \( E(\mathbf{r}) \) with \( M \)-dimensional isounits independent from the integration variables. The reader should be aware that in most applications of hadronic mechanics the isounits can be defined here, and they clearly do not constitute an isotopy.

As one can see, the initial and final parts of the lifting are elementary. The problem rest in the impossibility of achieving a consistent lifting of the intermediate step, that based on the partial derivative.

In the absence of a consistent isometry of the linear momentum, the early studies of hadronic mechanics lacked consistent formulations of physical quantities depending on the isometry, such as the isotope of angular momentum, kinetic energy, etc.

\[\phi (x) = \int (2/3)^{1/2} \int \text{d}t' (\frac{1}{2})^{1/2} \frac{\partial x^i}{\partial u^j} \text{d}u^j, \quad (3.287)\]

The origin of the above problem resulted in being where expected the least, in the ordinary differential calculus, and this explains the delay in the resolution of the impasse.

The above problem was finally resolved by Santilli in the second edition of Refs. [6,7] of 1996 (see Section 5.4.8 of Vol. I and Section 8.4.2 of Vol. II) with a mathematical presentation in memoir [10] of 1996. The resulting generalization of the ordinary differential calculus, today known as Santilli’s isodifferential calculus, plays a fundamental role for the study of the lifting equations by the first known structural generalization of Newton’s equations in Newtonian mechanics, and thus passing to the correct invariant formulation of all dynamical equations of hadronic mechanics.

For centuries, since its discovery by Newton and Leibniz in the mid 1600, the ordinary differential calculus had been assumed to be independent from the basic unit field and the same assumption was kept in the earlier studies on hadronic mechanics, resulting in the lack of full invariance, inability to formulate physical models and other insufficiencies.

After exhaust all other possibilities, an inspection of the differential calculus soon revealed that, contrary to an erroneous belief kept in mathematics for about four centuries, the ordinary differential calculus is indeed dependent on the basic unit field and related field.

In this section we review Santilli’s isodifferential calculus in its version needed for applications and verifications of hadronic mechanics. This update is recommendable because of various presentations in which the role of \( I \) and \( T \) were interchanged, resulting in possible ambiguities that could cause loss of invariance even under the lifting of the differential calculus.

A main feature is that, unlike all other aspects of hadronic mechanics, the isospace of the differential calculus cannot be resolved in the use of a nonconstant or nonunitary transforms, and have to be built via different, yet compatible methods.

Let \( S(\mathbf{r}, m, R) \) an N-dimensional metric or pseudo-metric space with contravariant coordinates \( R = (r^i) \), metric \( m = (m_{ij}) \), \( i, j = 1, \ldots, N \) and conventional unit \( I = \text{Diag}(1, \ldots, 1) \) on the real field. Let \( f(x) \) an ordinary (efficiently smooth) function on \( S \), let \( m^a \) be the flat metric tensor in the local coordinates, and let \( df(x)/dx^a \) be its partial derivative.

Let \( x^a = \text{m}^a x^i \), \( r^i = \text{m}^i x^a \), \( m^{ij} = (m_{ij})^{-1/2} \), \( (3.290a) \)

Let \( E(\mathbf{r}, M, R) \) be an isotope of \( E(\mathbf{r}) \) with \( M \)-dimensional isounits independent from the integration variables. The reader should be aware that in most applications of hadronic mechanics the isounits can be defined here, and they clearly do not constitute an isotopy.

As one can see, the initial and final parts of the lifting are elementary. The problem rest in the impossibility of achieving a consistent lifting of the intermediate step, that based on the partial derivative.

In the absence of a consistent isometry of the linear momentum, the early studies of hadronic mechanics lacked consistent formulations of physical quantities depending on the isometry, such as the isotope of angular momentum, kinetic energy, etc.
It is instructive for the reader interested in learning Santilli’s isodifferential calculus to prove that isoderivatives in different variables “commute” on isospaces over isofields,
\[
\frac{\partial f}{\partial \tilde{x}} = \frac{\partial f}{\partial \tilde{y}} = \frac{\partial f}{\partial \tilde{z}} = \frac{\partial f}{\partial \tilde{t}}.
\] (3.2.101)

But these properties on ordinary spaces over ordinary fields do not necessarily “commute”.

We are now sufficiently equipped to point out the completion of the construction of hadronic mechanics. First, let us verify the iso-pressing character of the isoequations of the isosequence in a covariant coordinate for the simple case in which the isoquant does not depend on the local variable. In fact, we have the expression
\[
\tilde{f}(\tilde{x}) = \tilde{f}(\tilde{y}) = \tilde{f}(\tilde{z}) = \tilde{f}(\tilde{t}) = \tilde{f}(\tilde{w}).
\]
(3.2.102)

Consider now the isopequation as a simply isode the conventional plane algebraic solution (again for the case in which the isoquant does not depend explicitly on the local coordinate),
\[
\tilde{f}(\tilde{x}) = \tilde{f}(\tilde{y}) = \tilde{f}(\tilde{z}) = \tilde{f}(\tilde{t}) = \tilde{f}(\tilde{w}).
\] (3.2.103)

for which we have the isoequations
\[
\frac{\partial \tilde{f}}{\partial \tilde{x}} = \frac{\partial \tilde{f}}{\partial \tilde{y}} = \frac{\partial \tilde{f}}{\partial \tilde{z}} = \frac{\partial \tilde{f}}{\partial \tilde{t}} = \frac{\partial \tilde{f}}{\partial \tilde{w}} = 0.
\]

We reach in this way the following fundamental definition of isosequence, first achieved by Santilli in Refs. [6, 7] of 1995, that completed the construction of hadronic mechanics (its insurance will be proved later on in Section 3.5).

DEFINITION 3.2.7 [6, 7]: The isobaric momenta on an iso-Hilbert space over the isofield of isocomplex numbers C (see Section 3.5 for details) is characterized by
\[
\frac{\partial \tilde{f}}{\partial \tilde{x}} = \frac{\partial \tilde{f}}{\partial \tilde{y}} = \frac{\partial \tilde{f}}{\partial \tilde{z}} = \frac{\partial \tilde{f}}{\partial \tilde{t}} = \frac{\partial \tilde{f}}{\partial \tilde{w}} = 0.
\]

(3.2.104)

Comparing the above formulation with Eq. (3.2.98), and in view of insurance (3.2.99), we reach the following

THEOREM 3.2.1 [6, 7, 10]: Planck’s constant h is the fundamental unit of the isodifferential calculus underlying quantum mechanics, i.e., quantum mechanical isocoordinates equations can be identically reformulated in terms of the isodifferential calculus with base isounit h.

\[
p \times \tilde{f}(\tilde{r}) = -i \hbar \times \frac{\partial \tilde{f}(\tilde{r})}{\partial \tilde{r}} = -i \hbar \times \frac{\partial \tilde{f}(\tilde{r})}{\partial \tilde{r}}.
\]
(3.2.105)

In conclusion, Santilli’s isodifferential calculus establishes that the isounit not only is the algebraic unit of hadronic mechanics, but also Planck’s constant with an interosequential operator \( \tilde{J} \), as would in represent constant, nonlinear, nonlocal and nonconservative effects.

More specifically, Santilli’s isodifferential calculus establishes that, while in exterior dynamical systems such as atomic structures, we have the conventional quantization of energy, in internal dynamical systems such as in the structure of hadrons, nuclei and stars, we have a superposition of quantized energy level at atomic distances plus continuous energy exchanges at hadronic distances.

Needless to say, all models of hadronic mechanics will be restricted by the condition
\[
\lim_{\tau \to 0} J \tau = \hbar,
\]
(3.2.106)
under which hadronic mechanics recovers quantum mechanics uniquely and identically.

DEFINITION 3.2.8 [6, 7, 11]: The isounits are defined by
\[
\tilde{e} = (-i) \hbar \frac{\partial}{\partial \tilde{r}} = \hbar \frac{\partial}{\partial \tilde{r}},
\]
(3.2.107)

while isocurrents are generated by
\[
\tilde{e} \tilde{f}(\tilde{r}) \cdot \frac{\partial \tilde{f}(\tilde{r})}{\partial \tilde{r}} = -\tilde{e} \tilde{f}(\tilde{r}) \frac{\partial \tilde{f}(\tilde{r})}{\partial \tilde{r}}.
\]
(3.2.108)

THEOREM 3.2.9 [6, 7, 11]: Isofields are isoeightful.

The latter new insurance constitutes an additional reason why the isodual theory of antimatter escaped attention during the 20th century.

3.2.6 Kadeisvili’s Isocontinuity and its Isodual

The notion of continuity on an isospace was first studied by Kadeisvili [19] in 1992 and it is today known as Kadeisvili’s isocontinuity. A review up to 1995 was presented in monographs [6, 7]. Rigorous mathematical study of isocontinuity has been done by Tagas and Semen [22, 23], B. M. Fomin Ganifilovna and J. Núñez Valdés [24, 26] and others. For mathematical studies we refer the interested reader to the latter papers. For the limited scope of this volume we shall present the notion of isocontinuity in its most elementary possible form.

THEOREM 3.2.6 [6, 7, 17]: The isodual isodierentials are defined by
\[
\frac{\partial \tilde{f}}{\partial \tilde{r}} = \lim_{\tilde{r} \to 0} \frac{\tilde{f}(\tilde{r}) + \tilde{f}(\tilde{r}) - \tilde{f}(\tilde{r})}{\tilde{r}}.
\]
(3.2.109)

with covariant version
\[
\frac{\partial \tilde{f}}{\partial \tilde{r}} = \lim_{\tilde{r} \to 0} \frac{\tilde{f}(\tilde{r}) + \tilde{f}(\tilde{r}) - \tilde{f}(\tilde{r})}{\tilde{r}}.
\]
(3.2.110)

It is then simple to reach the following

DEFINITION 3.2.6 [6, 7, 17]: The isodual isodierentials are defined by
\[
\frac{\partial \tilde{f}}{\partial \tilde{r}} = \lim_{\tilde{r} \to 0} \frac{\tilde{f}(\tilde{r}) + \tilde{f}(\tilde{r}) - \tilde{f}(\tilde{r})}{\tilde{r}}.
\]
(3.2.111)

with covariant version
\[
\frac{\partial \tilde{f}}{\partial \tilde{r}} = \lim_{\tilde{r} \to 0} \frac{\tilde{f}(\tilde{r}) + \tilde{f}(\tilde{r}) - \tilde{f}(\tilde{r})}{\tilde{r}}.
\]
(3.2.112)

It is then simple to reach the following

DEFINITION 3.2.6 [6, 7, 17]: The isodual isodierentials are defined by
\[
\frac{\partial \tilde{f}}{\partial \tilde{r}} = \lim_{\tilde{r} \to 0} \frac{\tilde{f}(\tilde{r}) + \tilde{f}(\tilde{r}) - \tilde{f}(\tilde{r})}{\tilde{r}}.
\]
(3.2.113)

with covariant version
\[
\frac{\partial \tilde{f}}{\partial \tilde{r}} = \lim_{\tilde{r} \to 0} \frac{\tilde{f}(\tilde{r}) + \tilde{f}(\tilde{r}) - \tilde{f}(\tilde{r})}{\tilde{r}}.
\]
(3.2.114)

It is then simple to reach the following

DEFINITION 3.2.6 [6, 7, 17]: The isodual isodierentials are defined by
\[
\frac{\partial \tilde{f}}{\partial \tilde{r}} = \lim_{\tilde{r} \to 0} \frac{\tilde{f}(\tilde{r}) + \tilde{f}(\tilde{r}) - \tilde{f}(\tilde{r})}{\tilde{r}}.
\]
(3.2.115)

with covariant version
\[
\frac{\partial \tilde{f}}{\partial \tilde{r}} = \lim_{\tilde{r} \to 0} \frac{\tilde{f}(\tilde{r}) + \tilde{f}(\tilde{r}) - \tilde{f}(\tilde{r})}{\tilde{r}}.
\]
(3.2.116)

THEOREM 3.2.6 [6, 7, 17]: Isocontinuities are isocontinuous.
The isodual invariance is a simple isodual image of the preceding notion of continuity and will be herein assumed.

3.2.7 TSSFN Isotopology and its Isodual

Topology is the ultimate foundation of quantitative sciences because it identifies on rigorous mathematical grounds the limitations of the ensuing description. Throughout the 20th century, all quantitative sciences, including particle physics, nuclear physics, astrophysics, superconductivity, chemistry, biology, etc., have been restricted to the use of mathematics based on the conventional local-differential topology, with the consequence that the sole admitted representations are those dealing with a finite number of isolated point-like particles. Hence points are dimensions, they cannot have contact interactions. Therefore, an additional consequence is that the sole possible interactions are those of action-at-a-distance type representable with a potential.

In conclusion, the very assumption of the conventional local-differential topology, such as the conventional topology for the Euclidean space, or the conventional topology for the Minkowski space, uniquely and unambiguously restrict the admitted systems to be local, differential and Hamiltonian. The provided apprehension of systems that proved to be excellent whenever the mutual distances of particles are much greater than their size as it is the case for planetary and atomic systems.

However, the above conditions are the exception and not the rule in nature, because all particles have a well defined extended wavelength and/or charge distribution of the order of $10^{-13}$ m. It is well known in pure and applied mathematics that the representation of the actual shape of particles is impossible with a local-differential topology.

Moreover, once particles are admitted as being extended, there is the emergence of the additional contact, zero-range nonpotential interactions that are nodocal in the sense of occurring in a finite surface or volume that cannot be consistently reduced to a finite number of isolated points.

Consequently, it is equally true by experts that conventional local-differential topologies cannot represent extended particles at short distances and their nonlocal-nonpotential interactions, as expected in the structure of planets, strongly interacting particles, nuclei, molecules, stars and other interstellar dynamical systems.

The need to build a new topology, specifically conceived and constructed for hadronic mechanics was suggested since the original proposal [2] of 1979. It was not only until 1995 that the Greek mathematician Gr. Tsagias and D. S. Svorlas [22,23] proposed the first isotopological on acoustical record formulated on iso-spaces over ordinary fields. In 1996, the Italian-American physicist R. M. Santilli [10] extended the formulation to isospaces over isofields. Finally, comprehensive studies on isotopology were conducted by the Spanish Mathematicians R. M. Falcon Ganfornina and J. Nu~nez Valdes [24,25]. As a result, the new topology is here called the Tsagas-Sourlas-Santilli-Falcon-Nu~nez isotopology (or TSSFN Isotopology for short).

The author has no words to emphasize the far reaching implications of the new TSSFN isotopology because, for the first time in the history of science, mathematics can consistenly represent the actual extended, generally nonelastic and deformable shape of hadrons, their densites as well as their nonpotential and nonlocal interactions.

As an example, Newton's equations have remained unchanged in Newtonian mechanics since the time of their conception to represent point-particles. No consistent generalization was possible due to the underlying local-differential topology and related differential calculus. As we shall see in the next section, the nondifferential calculus and underlying isotopology will permit the first known structural generalization of Newton's equations in Newtonian mechanics for the representation of extended particles.

New coverings of quantum mechanics, quantum chemistry, special relativity, and other quantitative sciences are then a mere consequence. Perhaps more importantly, the new clean energies and fuels permitted by hadronic mechanics can see their origin precisely in the TSSFN isotopology, as we shall see later on in this chapter.

In their most elementary possible form accessible to experimental physicists, the main lines of the new isotopology can be summarized as follows. Being numbers seminal, Hermitian and positive-definite, N-dimensional isocartesian can always be diagonalized into the form

$$\bar{I} = \bar{D}_0 \bar{a}_1 \ldots \bar{a}_n \bar{m} \ldots \bar{a}_1 \bar{D}_0 = \bar{I} \quad \text{(3.113)}$$

Consider N normal isofields $\bar{b}_k(i \cdot \lambda)$ each characterized by the isoinv $\bar{b}_k = a^k_i$ with (ordered) Cartesian product $\bar{b}^0 = \bar{b}_1 \times \bar{b}_2 \times \ldots \bar{b}_2$.

Since each isofield $\bar{b}_k$ is isomorphic to the conventional field of real numbers $\mathbb{R}(i \cdot \lambda)$, it is evident that $\bar{b}^0$ is isomorphic to the Cartesian product of $N$ ordinary fields $\mathbb{R}^N = \mathbb{R} \times \mathbb{R} \times \ldots \mathbb{R}$. (3.115)

Let $\tau = (\bar{b}, \bar{k})$ be the conventional topology on $\mathbb{R}^N$ (whose knowledge was hence assumed for brevity), where $\bar{b}$ represents the subset of $\mathbb{R}^N$ defined by

$$\bar{k} = \{ \bar{p} = (\bar{a}_1, \ldots, \bar{a}_n) | \bar{a}_1, \ldots, \bar{a}_n \leq C; \bar{m}, \bar{n}, \bar{a}_1, \ldots, \bar{a}_n \in \mathbb{R} \} \quad \text{(3.117)}$$

\[ \text{Figure 3.6. A schematic view of the "isospace," namely, the perfect sphere on isospace over isofield represented by isoinv (3.2.121), that is assumed to be the geometric representation of hadrons used in this monograph. The actual nonpotential and deformable shape of hadrons is obtained by projecting the isospace in our Euclidean space, as illustrated in the last identity of Eq. (1.2.14).} \]

The new TSSFN isotopology is equipped with a vectorcr structure, an isocartesian structure and the mapping

$$\bar{F}: \mathbb{R}^N \rightarrow \bar{b} \quad \bar{a} \rightarrow \bar{f}(\bar{a}), \forall \bar{a} \in \mathbb{R}^N \quad \text{(3.2.118)}$$

An iso-Euclidean isomorphoid $\mathbb{M}(\bar{F} : \bar{b}, \bar{k})$ occurs when the N-dimensional isospace $\bar{b}$ is realized as the Cartesian product (3.2.106) and equipped with isotopology (3.2.118) with base isomorphoid (3.2.113).

The new TSSFN isotopology and related notions can easily be constructed with the isodual map (3.2.15) and its explicit study is left as an interesting exercise for the interested reader.

3.2.8 Iso-Euclidean Geometry and its Isodual

The isoduals of the Euclidean space and geometry were introduced for the first time by Santilli in Ref. [14] of 1983 as a particular case of the broader isoduals of the Minkowski space and geometry treated in the next section.

The same isoduals were then studied in various works by the same author and the comprehensive treatment was presented in Chapter 5 of Vol. 1 [19]. These isoduals are today known as the Euclid-Santilli isospace and isoperometry. The presentation of Vol. 1 will not be repeated here for brevity. We merely limit ourselves to outline the main aspects for minimal self-sufficiency of this monograph.

Consider the fundamental isospace for nonrelativistic hadronic mechanics, the three-dimensional Euclid-Santilli isospace with isometric invariance $\bar{r}$ and
isometric \ddot{\ell} over the isosurface \(R = R(\ell, \dot{\ell}, \ddot{\ell})\) (see Section 3.3)
\[E(\ell, \dot{\ell}, \ddot{\ell}) = \ddot{\ell}^2 + (\dot{\ell}_1^2 + \dot{\ell}_2^2 + \dot{\ell}_3^2)\]

It then follows that Euclidean space is not necessarily preserved under isotopies.

Note that, while the Euclidean space and geometry are unique, there exist an infinite family of different yet isomorphic Euclid-Santilli isospaces and isogeometries, evidently characterized by different isometrics in three dimension and signature \((\pm, \pm, \pm)\).

Recall from Section 3.2.3 that the structure of the basic invariant is given by Eq. (3.2.66). Therefore, the isosepse, namely, the image on \(R\) of the perfect sphere on \(\tilde{R}\) remains a perfect sphere. However, the projection of the isosepse on the original space \(R\) is a spherical ellipsoid, as clearly indicated by invariant (3.2.121).

Therefore, the isosepse on an isosurface unifies all possible spherical ellipsoids on ordinary spaces over ordinary fields. These features are crucial to understand later on the reconstruction of the exact rotational symmetry for deformed spheres (see Fig. 3.7).

Since the functional dependence of the isometric is unrestricted except verifying the condition of point-definition, it is easy to see that the Euclid-Santilli isometry unifies all possible three-dimensional geometries with the signature \((\pm, +, +)\), thus including as particular cases the Riemannian, Finslerian, non-Darbouxian and other isometrics. As an example, the Riemannian metric \(g_{\mu\nu}(r) = g_\mu r\) is a trivial particular case of Santilli’s isometric \(\tilde{g}_{\mu\nu}(r, \ldots)\). This occurrence has profound physical implications that will be pointed out in Section 3.5.

Yet another structural difference between conventional and isometric geometries is that the former has the same unit for all three reference axes. In fact, the geometric unit \(R = Diag(1, 1, 1)\) is a dimensionless representation of the selected units, for instance, \(R = \text{Diag}(1 \text{ cm}, 1 \text{ cm}, 1 \text{ cm})\). In the transition to the isospace, the unit is different for different axes and we have, for instance, \(R = \text{Diag}(1 \text{ cm}, 1 \text{ cm}, 1 \text{ m})\). It then follows that shapes detected by our sensory perception are not necessarily absolute, in the sense that they may appear basically different for an isotope observer (see Fig. 3.7).

is a more consequence of our particular sensory perception, with different dimensions occurring for other observers.\(^{12}\)

The occurrence was discovered by Santilli in Ref. [6], page 213, via the following isotopic element
\[T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \]

is positive definite since Det \(T = 1\); thus being a fully acceptable isotopic element.

It is easy to see that the isoinvariant of the Euclid-Santilli isosurface characterized by the above non-diagonal isotopy is given by
\[\tilde{\ell}^2 = \tilde{v}^2 + \tilde{a}_1^2 - \tilde{a}_2^2 - \tilde{a}_3^2
\]


\[\Delta = \delta \times \delta = \text{Diag}(n_1^2, n_2^2, n_3^2)\]

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3.2.9 Minkowski-Santilli Isogonomy and its Isodual

3.2.9A. Conceptual Foundations. The isogonies of the Minkowskian and Santilli geometries are the main mathematical methods of relativistic hadronic mechanics, because they are at the foundations of the Poincare-Santilli isogonomy, and related broadening of special relativity for relativistic internal dynamical systems.

The isogonies of the Minkowskian and Santilli geometries were first proposed by Santilli in Ref. [14] of 1983 and then studied in numerous papers (see monographs [6,7,14,15] and papers quoted therein) and are today known as Minkowski-Santilli isogeometries and isomathematics.

Due to their fundamental character, the new spaces and geometries were treated in great details in Ref. [6,7], particularly in the second edition of 1995, and that presentation is here assumed as known for brevity.

The primary purpose of this section is to identify the most salient advances occurred since the second edition of Refs. [6,7] with particular reference to the geometric treatment of gravitation.

To ensure the original efforts in the construction of relativistic hadronic mechanisms were based on two different isogeometries, the isogonies of the Minkowskian geometry for gravitational aspects. The presentation of Refs. [6,7] was based on the isotopies of the Minkowskian and Riemannian geometries for nongravitational profiles, and the isotopies of the Riemannian geometry admits, as a particular cases, all possible Riemannian metrics.

The isotopies of the Minkowskian and geometry were first proposed by Santilli in Ref. [14] of 1983 and then studied in numerous papers (see monographs [6,7,14,15] and papers quoted therein) and are today known as Minkowski-Santilli isogeometries and isomathematics.

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transformation, thus implying the majestic mathematical and physical consistency of special relativity recalled in Chapter 1, while the time evolution of the latter is a noncanonical transform, thus implying a number of unresolved problematic aspects that have been lingering throughout this century.

The reformulation of the Riemannian geometry in terms of the Minkowski ansatz is the sole possibility known to the author for achieving automatic consistency under a nontrivial functional dependence of the metric.

In summary, Minkowski ansatz have the following primary applications. First, they are used for a re-interpretation of the Riemannian metrics $g(x)$ for the particular case $g = (g(x) = g(x))$ (3.140) which is the fundamental expression for variances. Second, the same isoperme are used for the characterization of inter-space gravitational problems with isometries of unrestricted functional dependence.

$\tilde{g} = (\tilde{g}(x, v, \nu, \rho, \ldots) = \tilde{g}(x, v, \nu, \rho, \ldots)$ (3.147)

while preserving the original Minkowski ansatz.

Since the explicit functional dependence is essential under isoperme, our studies will be generally referred to the interior gravitational problem. Unless otherwise stated, only diagonal realizations of the isoperme will be used hereon for simplicity. An example of non-diagonal isoperme inherent in a structure proposed by Drac is indicated in Section 3.5. More general liftings of the Minkowski space of the so-called genotypic and multirelational hyperspace type will be indicated in Chapter 4.

3.2.9C. Indeterminate, Inconsistent, and Isolativeness. In the preceding subsections we have presented the Minkowski aspects of the new isoperme. We are now sufficiently equipped to present the novel part of the Minkowski-Santilli isoperme, its Riemannian characer as first derived in Ref. [26].

Our study is strictly in local coordinates representing the fixed frame of the observer without any un-needed use of the transformation theory or abstract treatments. Our presentation will be as elementary as possible without reference to advanced topological requirements, such as Kolmogorov’s incompatibility (Section 3.2.6), nonsimultaneous and related TNPSN relativity (Section 3.2.7).

Also, our presentation is made, specifically, for the $(0+1)$-dimensional isoperme, with the understanding that the extension to arbitrary dimensions and signatures or signatures different than the conventional one $(x, \ldots, v, \ldots)$ is elementary, and will be left to interested readers.

Let $M(\mathbf{z}; G; R)$ be a Minkowski-Santilli isoperme and let $\hat{M}(\mathbf{z}; G; R)$ be its projection in our spacetime as per Definition 3.12. To illustrate the transition from isocomodized $\mathbf{z}$ to conventional spacetime coordinates $x$, we shall denote

$$\mathbf{z} = \hat{\mathbf{z}} = \hat{\mathbf{z}}(x, y, \nu, \rho, \ldots)$$

which illustrates more clearly the presentation under the dual lifting $\eta = \eta = T$, $\eta = \eta = T$ of the original ansatz as well as numerical values.

**Theorem 3.39 3.39 [9,3.26]:** Conventional and isopicetic isopermeties of spacetime are $n$-dimensional.

**Proof.** In addition to the $n$-dimensionality of the Poincaré symmetry, there is an additional $n$-th dimensionality characterized by the isopermet $g = g = n^{n^T}$, $I = I = n^{I}$.

$$(3.144)$$

**Note.** The crucial role of Santilli's isocomodizers in the above property. The explains why the $11$-th dimensionality remained undiscovred throughout the 20th-century.

A significant difference between the conventional $n$ and its isomod $\eta$ is that the former admit only one formulation, the conventional one, while the latter admit two formulators; one on isospace itself, i.e., expressed with respect to the isospace $I$ and its projection in the original space $M$ (i.e., expressed with respect to the conventional unit $E$).

Note that the projection of $M(\mathbf{z}; G; R)$ into $M(\mathbf{z}; G; R)$ is not a conformal map, but an inverse isopicetic map because it implies the transition from generalized units and fields to conventional units and fields.

The axonomic motivation for constructing the isopermetes of the Minkowskian geometry is that any modification of the Minkowskian metric requires the use of noncanonical transforms $x \rightarrow x^p$, (3.145)

$$(3.145)$$

and this includes the case of the transition from the Minkowskian metric $g$ to the Poincaré-Santilli metric $g$.

In turn, all noncanonical theories, thus including the Riemannian geometry, do not possess invariant units of space and time, thus having the characteristic incompatibilities studied in Chapter 1. A primary axonomic function of the isospace is that of restoring the invariance of the basic units, as established by the Poincaré-Santilli isocomodizer.

This is achieved by embedding all noncanonical content in the generalization of the unit. Invariance for noncanonical structures such as Riemannian metric is then assured by the fact indicated earlier that, whether conventional or generalized, the unit is the basic invariant of any theory. Stated in different terms, a primary axonomic difference between the special and general relativities is that the time evolution of the former is a conventional
\[ P^\alpha_{\beta\gamma} = \Phi^\alpha_{\beta\gamma} \]  

(3.2.1026)

Note the unrestricted functional dependence of the connection which is notably absent in conventional treatments. Note also the abstract identity of the conventional and isotopic connections. Note finally that local numerical values of the conventional and isotopic connections coincide when computed in their respective spaces. This is due to the fact that in Eq. (3.2.152) \( \Phi^\alpha_{\beta\gamma} \) for exterior problems, while the value of derivatives \( \partial \) and modernisation \( \partial^\alpha \) coincide when computed in their respective spaces.

Note however that, when projected in the conventional spacetime, the conventional and isotopic connections are different even in the exterior problem in which \( \Phi = \Phi_{\beta\gamma} \),

\[ \Gamma_{\alpha\beta\gamma} = \frac{1}{2} \left( \partial^\alpha \Gamma_{\beta\gamma} + \partial^\beta \Gamma_{\alpha\gamma} - \partial^\gamma \Gamma_{\alpha\beta} \right) \times \partial^\alpha. \]  

(3.2.153)

The extension to covariant vector fields and covariant or contravariant vector fields is consequential.

Without proof we quote the following important result from Ref. [26]:

**LEMMA 3.2.5 (Iso-Ricci Lemma) [26]**. Under the assumed conditions, the invariant derivatives of all isometrics in Minkowski-Santilli isospace spaces are identically null,

\[ \Phi^\alpha_{\beta\gamma} = 0, \quad \alpha, \beta, \gamma = 1, 2, 3, 4. \]  

(3.2.154)

The novelty of the isometry is then illustrated by the fact that the Ricci property persists under an arbitrary dependence of the metric, as well as under Minkowski, rather than Riemannian axes.

The isotensors on \( M \) is defined by

\[ \Phi^\alpha_{\beta\gamma} = \Phi^\alpha_{\beta\gamma} - \Phi^\alpha_{\gamma\beta}, \]  

(3.2.155)

and coincides again with the conventional torsion at the abstract level, although the two torsions have significant differences in their explicit forms when both projected in our space-time.

**DEFINITION 3.2.14 [26]**. The Minkowski-Santilli isoscalar is characterized by the following isotensor: the isoscalar isotensor

\[ R^\alpha_{\beta\gamma} = \Phi^\alpha_{\beta\gamma} - \Phi^\alpha_{\gamma\beta} + \Phi^\alpha_{\delta\gamma} I^\gamma_{\beta\delta} - \Phi^\alpha_{\delta\beta} I^\gamma_{\gamma\delta}. \]  

(3.2.156)

the iso-Ricci isotensor

\[ R^\alpha_{\beta\gamma} = R^\alpha_{\gamma\beta}. \]  

(3.2.157)

\[ \Phi^\alpha_{\beta\gamma} \]  

is the isoscalar isotensor

\[ R^\alpha_{\beta\gamma} = R^\alpha_{\gamma\beta}, \]  

(3.2.161)

and the need for a source term also in exterior gravitation in vacuum mandated by the Freud identity and other reasons

\[ R^\alpha_{\beta\gamma} = \tilde{R}^\alpha_{\beta\gamma} + \tilde{R}^\alpha_{\gamma\beta}. \]  

(3.2.162)

Fried's identity was rediscovered by the author during his accurate study of Pauli's historical book and studied in detail in Refs. [67, 77] of 1992. Additional studies of the Freud identity were done by Yilmaz [30]. Following a suggestion by the author, the late mathematician Ennio Rund [29] studied the identity in one of his last papers and proved that:

**LEMMA 3.2.8 (Rund's Lemma) [29]**. Freud's identity is a bona fide identity for all Riemannian spaces irrespective of dimension and signature.

In this way, Rund confirmed the general need of a source also in vacuum (see Sections 1.4 and 3.5).

Following Ref. [26], in this paper we have presented the isotopes of the Freud identity on Minkowski-Santilli isospace, as characterized by the isoscalar calculus. Its primary functions for this monograph is to identify the geometric structure of the interior gravitation problem. The presence of the source in vacuum is per the Freud identity, electrodynamics and other needs will then be consequential, thus confirming the inconsistencies of Einstein's conception of gravity in vacuum as pure curvature without source.

Note that all conventional and isotopic identities coincide at the abstract level. 3.2.9E. Isoparallel Transport and Isosemigroups. An isovector field \( X^\alpha \) on \( M = \mathcal{M} (\mathcal{M}, \mathcal{R}) \) is said to be transported by isoparallel displacement from a point \( m(z) \) on a curve \( C \) to a neighboring point \( \tilde{m}(z + \delta z) \) on \( C \),

\[ DX^\alpha = X^\alpha + \tilde{X}^\alpha + \tilde{X}^\alpha \delta \zeta. \]  

(3.2.168)
(3.2.170)

where one should note the isotopic character of the integration. The isotopy of the conventional case then yields the following:

**LEMMA 3.2.9** [36]: Necessary and sufficient conditions for the existence of an isoparallel transport along a curve $C$ on a $(3+1)$-dimensional Minkowski-Santilli isospace is that all the following equations are identically verified along $C$

$$\nabla_C \dddot{x} = 0, \quad \beta, \gamma = 1, 2, 3, 4$$

(3.2.171)

Note, again, the abstract identity of the conventional and isotope parallel transport. However, it is easy to see that the projection of the isoparallel transport in ordinary spacetime is structurally different than the conventional parallel transport.

Consider, as an example, a straight stick partially immersed in water. It is an instructive exercise for the interested reader to construct the classical representation of such an isoparallel transport in isospace.

The simplest possible example is given by the isoeuclidean representation of a straight stick partially immersed in water. In conventional representations the stick penetrating in water with an angle $\alpha$ appears as bended at the point of immersion in water with an angle $\gamma = \alpha + \beta$, where $\beta$ is the angle of refraction. In an isoeuclidean representation the stick remains straight along isospace because the isoscalar $\gamma = \gamma \hat{I}$, whereas the original angle $\hat{I} = \alpha + \beta$. The situation is essentially the same for our representation of interior gravitation because the latter is represented in isospace over isofields via isofields equations. However, it is known that forms coincide with conventional equations on a conventional Riemannian spacetime. Being noninvariantly, all interior features are invariantly represented via generalized units.

3.2.9F. Isoindal Minkowski-Santilli isosphaces and isogeometry. The isosbold Minkowski-Santilli isosphaces were introduced for the first time by Santilli in Ref. [8] of 1965 and then studied in various works (see the references of Chapter 1), and can be written

$$\delta^I = \delta^I(x^0, x^1, \ldots, x^3)$$

(3.2.174a)

$$\delta^I = \delta^I(x^0, x^1, \ldots, x^3)$$

(3.2.174b)

The isosbold Minkowski-Santilli isosbold isometry is the geometry of isosbold isosphaces $M^4$ over $R^4$ and isosbold for the first time by Santilli in Ref. [26] of 1968. The isosbold Minkowski-Santilli isosbold isometry has the geometry of isosbold isosphaces $M^4$ over $R^4$ and was studied for the first time by Santilli in Ref. [26] of 1968. The physically and mathematically most salient property of the latter geometry is that it is characterized by isosbold units of space, time, etc., and isometric volumes. Therefore, in addition to a change in the sign of the charge, we also have change of sign of masses, energies, and other quantities positively and negative for matter. Similarly, we have the isosbold isosphere and isometric coordinates in the isospace representation of isosbold in isospace over isofields.

Thus, motion under isosbold isometry is in a time direction opposite to the conventional motion. Those features are necessary as to have a classical representation of antimatter in interior conditions whose operator image yields indeed antiparticles (rather than particles with the wrong sign of the charge).

We also have the following important

**LEMMA 3.2.12** [37]: Isoinertial flows are independent from isospace inertions

$$\vec{r} = x = x = -\vec{r}, \quad \vec{r} = t \times \vec{x} = -\vec{r}$$

(3.2.178)
following isomorphisms:

Isodual isomorph:
\[ i \rightarrow i', \quad \theta \rightarrow \theta', \quad \omega \rightarrow \omega', \quad \mathcal{H} \rightarrow \mathcal{H}' \]

Isodual isosymplectic two-form:
\[ \Omega = \theta \wedge \omega = \theta' \wedge \omega' = \mathcal{H} \wedge \mathcal{H}' \]

Isodual iso-differential:
\[ d = \partial = \partial', \quad \partial_r \rightarrow \partial_{r'} = \partial_{r'} = \partial_r \]

Isodual iso-integrals:
\[ \int \rightarrow \int', \quad \int_\mathcal{V} = \int_\mathcal{V}' = \int_\mathcal{V} \]

Isodual iso-tensors:
\[ \mathcal{A}_r = \mathcal{A}_{r'} = \mathcal{A}_{r'}, \quad \mathcal{G}_{r;p} = \mathcal{G}_{r;p'} = \mathcal{G}_{r;p} \]

Isodual iso-Ricci isotensor:
\[ \mathcal{R}_{r;r} = \mathcal{R}_{r;r'} = \mathcal{R}_{r;r} \]

Isodual iso-electromagnetic potentials:
\[ A_r = A_{r'} = A_{r'}, \quad \mathcal{E}_r = \mathcal{E}_{r'} = \mathcal{E}_{r} \]

We now consider the isosymplectic and isodifferential geometry, and consider for clarity and simplicity only its realization in a local chart (rather than quantization).

As it is well known, the isounit, called the isounit, is the object of a fundamental isounit, called the isounit, that, under the assumption for simplicity that the isounit is positive-definite, is given by:
\[ \mathcal{H} = \mathcal{H}' = \mathcal{H} = \mathcal{H}' = 1 \]

The fundamental isosymplectic two-form is then given by:
\[ \omega = \theta \wedge \omega = \theta' \wedge \omega' = \mathcal{H} \wedge \mathcal{H}' \]

The fundamental isotopies then follow from the assumption that the isounit is positive-definite, and therefore:
\[ \mathcal{H} = \mathcal{H}' = \mathcal{H} = \mathcal{H}' = 1 \]

The fundamental isocanonical two-forms are then given by:
\[ \omega = \theta \wedge \omega = \theta' \wedge \omega' = \mathcal{H} \wedge \mathcal{H}' \]

The fundamental isosymplectic geometry can be considered mature unless it ad-

\[ \mathcal{H} \rightarrow \mathcal{H}' = \mathcal{H} \rightarrow \mathcal{H}' \]

The fundamental isosymplectic form is then given by the exterior derivative of the following isomorphisms:
\[ \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} \]

The fundamental isosymplectic form is then given by the anterior derivative of the following isomorphisms:
\[ \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} \]

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\[ \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} \]

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\[ \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} = \frac{d\mathcal{H}}{\mathcal{H}} \]

As we shall see better in Section 3.3, direct universality then follows from the assumptions of the fundamental isounit and its implications.
3.2.11 Isolinearity, Isocollinearity, Isocanonicity and Their Isoolonalities

In Section 3.1 we pointed out that the primary physical characteristics of particles and antiparticles in interior conditions (such as a neutron in the core of a neutron star) are nonlinear, nonlocal and noncanonical due to the mutual penetration-overlapping of their wavepackets with those of the surrounding medium.

In the preceding subsections we have identified isotopic means for mapping linear, local and canonical systems into their most general possible nonlinear, nonlocal and noncanonical form. In this section we show how the isotopes permit the reconstruction of linearity, locality and canonicity on iso spaces over iso fields, called linearity, locality and canonicity for the case of particles, with their iso collinear counterpart for antiparticles.

The understanding of this seemingly impossible task requires the knowledge that conventional methods have only one formulation. By contrast, all isotopic methods have a dual formulation, the first in iso spaces over iso fields, and the second when projected in ordinary spaces over ordinary fields. Deviations from conventional properties can only occur in the latter formulation because in the former all isolated atomic properties are preserved by construction.

Let $S(R) = \mathbb{R}$ be a conventional real vector space with local coordinates $r$ over the real $\mathbb{R} = \langle n, \theta, \phi \rangle$, and let

$$ r' = A(r) \cdot r, \quad r'' = r' \times A'(r), \quad w = w' $$

be a conventional right and left linear, local and canonical transformation on $S$, where $t$ denotes transpose.

The isotopic lifting $S(R) = S(\mathbf{R})$ requires a corresponding necessary iso theory of the transformation theory. In fact, it is instructive for the interested reader to verify that the application of conventional linear transformations to the iso space $S(\mathbf{R})$ causes the loss of linearity, transitivity and other basic properties.

For these and other reasons, Santilli submitted in the original proposals [4, 5] of 1978 (see monographs [6, 7], for comprehensive treatments and applications) the isotopy of the transformation theory, called isotransformation theory, which is characterized by isotransforms (where we make use of the notion of isoduction of Section 3.2.4)

$$ r' = A(r) \cdot r, \quad r'' = r' \times A'(r), \quad w = w' $$

where $R$ denotes the resulting transformation on $S$, which is the foundation of the direct universality of isotopic methods, that is, their applicability to all possible (sufficiently smooth and regular) nonlinear, nonlocal and noncanonical systems (universality) in the frame of the experimenter (direct universality).

In order to apply isotopic methods to a nonlinear, nonlocal and noncanonical system, one must have already identified one of its possible isolinear, isolocal and isocanonical isounits in the system of coordinates. The applicability of the methods studied in this monograph then follows.

The isotopic isounit (3.1.9) is given by the group of isotransforms (3.1.20) under isounitarity, and, as such, are defined on the iso space $S(R, \mathbf{R})$ over the isotopic iso space $\mathbb{R}$ with iso unit $I' = 1/\mathbb{R}' = I = [6, 7]$ with evident properties

$$ A' \circ A'' = A'' \circ A', \quad A' \circ A'' = A'', \quad A' \circ I' = I', \quad A' \circ A'' = A'' \circ A' $$

The definition of iso unitarity, isounitarity and isoollonality then follows.

From now on, we will use isounit transformations for the study of interior dynamical systems of particles and their isounit for interior systems of antiparticles.

3.2.12 Lie-Santilli Isotropy and its Isoideal

3.2.12A. Statement of the Problem. As it is well known, Lie’s theory has permitted outstanding achievements in various disciplines throughout the 20th century. Nevertheless, in its current conception and realization, Lie’s theory is linear, local, differential and canonical-Hamiltonian.23

The author has proposed for over a decade that mathematicians use the property of this Corollary 3.2.15A to identify the simplest methods for the solution of nonlinear differential equations, but the request has not yet been met as yet, to our best knowledge.

Yet another need in physics is the identification of the exact symmetry that can effectively replace Lie symmetries, which symmetry cannot be a conventional Lie symmetry due to the need of preserving the original dimensional symmetries in order to avoid the prediction of nonphysical effects and/or hypothetical new particles.

It is evident that Lie’s theory in its current formulation is solely applicable to matter, evidently because there exists no antiformomorph version of the conventional Lie’s theory as necessary for the correct treatment of antiamatter beginning at the classical level, as shown in Chapters I and 2.

Another central problem addressed in these studies is the construction of the universal symmetry (and not “convariance”) of gravitational matter and, independently, for antimatter, that is, a symmetry for all possible external and interior gravitational line elements of matter and, under antichromatic image, of antimatter.

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As such, Lie’s theory is exactly valid for exterior dynamical systems, but possesses clear limitations for interior dynamical systems since the latter are nonlinear, nonlocal and noncanonical. This occurrence mandates a suitable version of Lie’s theory such to be exactly valid for interior dynamical systems without approximations.

Independently from that, Lie’s theory in its current formulation is solely applicable to matter, evidently because there exists no antiformomorph version of the conventional Lie’s theory as necessary for the correct treatment of antiamatter beginning at the classical level, as shown in Chapters I and 2.

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Another central problem addressed in these studies is the construction of the universal symmetry (and not “convariance”) of gravitational matter and, independently, for antimatter, that is, a symmetry for all possible external and interior gravitational line elements of matter and, under antichromatic image, of antimatter.
the transition from closed Hamiltonian and non-Hamiltonian systems (see Sec-
3.1.2). Equivalently, the generators of Lie's theory cannot be altered by
non-Hamiltonian effects.

This physical requirement can only be achieved by preserving conventional
generators $X_0$ and lifting instead their product $X_0 X_j = X_j X_0 = X_j + X_0$, which
is the original formulation of the Lie-Santilli isochromism [6] and remain the
formulation needed for applications to this day. It is essentially given by the
projection of the isotopic formulation on conventional spaces over conventional
fields.

3.2.12C. Lie-Santilli Isoalgebras.

As it is well known, Lie algebras are the antisymmetric algebras $L = [[L]]$:
attached to the universal enveloping algebras $(L)$. This main characteristic is preserved although enlarged under isotopes (see (2.2) for details).
We therefore have the following:

**DEFINITION 3.2.15 [4]**: A finite-dimensional isospace $L$ with generic
elements $A, B, \ldots$, over the isofield $F$ with isoval $1 = 1_F > 0$ is called a
Lie-Santilli isospace over $F$ when there is a composite $[A, B]$ in $L$, called
"isooperator", that is indistinguishable in an isospace and such that all the following
axioms are satisfied:

\[
[A, B] = [B, A], \quad [A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0, \quad [A, [B, C]] = [A, B] [B, C] + [B, A] [C, B].
\]

The isosystems are said to be: "isoscalar, isospin or isospace-isosystem depending
on the assumed isofield and isospace when $[A, B] = 0, B, C \in L$. A subset $L'$ of
$L$ is said to be an isosubalgebra of $L$ when $[X, Y] \in L'$, $X, Y \in L$. A maximal
isoval $\geq 0$ verifying the property $[X, Y] = 0$ is called the isovelvet of $L$.

For the isotopes of additional conventional notions, theorems and properties of
Lie algebras, one may see monographs [2,6,36,37].

We merely recall the isotope generalizations of the celebrated Lie's First, Sec-
ond and Third Theorems introduced in the original proposal [4], but which we do not
review here for brevity. For instance, the Lie-Santilli Second Isoisomorphism
rules:

\[
[X, Y] = X \cdot Y - Y \cdot X = [X, Y] + [Y, X] = 0.
\]

\[
[X, [Y, Z]] = X \cdot [Y, Z] - [Y, Z] \cdot X = [X, Y] [Y, Z] + [Y, X] [Z, Y] + [Z, Y] [X, Y],
\]

are the following isosymmetric identities.

We study the isosymmetric properties of the Lie-Santilli isoalgebras $L$ in a
number of contexts where the isoelements are related to corresponding
isopermutations $G$ and vice-versa. Lie transformation groups $G$ admit their
corresponding Lie objects $l$ when computed in the neighborhoods of the identity $I$
and $G$.

The basic properties are preserved under isotopes although broadened to the
most general possible nonlinear, nonlocal and noncanonical transformations

**DEFINITION 3.2.16 [4]**: A right isomodular Lie-Santilli transformation

\[
(x, y, \ldots) \rightarrow (x', y', \ldots) \subseteq S \text{ into a new element } x' \in S
\]

is called an isomodular Lie-Santilli transformation $\phi$. All such transformations

\[
\phi : \phi(x, y, \ldots) \in \tilde{S}, \quad \phi(\phi(x, y, \ldots)) \in \tilde{S}
\]

that such:
with infinitesimal version in the neighborhood of $\mathbf{i}$

\[ A_{\mathbf{i}}(d\mathbf{a}) = (\mathbf{g} + d\mathbf{a} + \mathbf{X} \cdot x + \ldots) \cdot A(\mathbf{g} + \mathbf{a} + \mathbf{X} \cdot x + \ldots) = \]

\[ = A(\mathbf{g}) + d\mathbf{a} + \mathbf{X} \cdot x + \ldots \quad \text{(3.2.22)} \]

that can be written

\[ \mathbf{X} \cdot d\mathbf{a} = \mathbf{A} \cdot (\mathbf{X} \cdot d\mathbf{a}) = [\mathbf{A}, \mathbf{X}] \quad \text{(3.2.22)} \]

Note the crucial appearance of the isometric element $T(x, z, \ldots)$ in the exponent

of the isogroup. This ensures a structural generalization of Lie's theory of the
desired nonlinear, nonlocal and nonconventional form.

Still another important property is that conventional group-composition laws
admit a consistent isogroup lifting, resulting in the following Baker-Campbell-
Hausdorff-Striuli isometrization

\[ (\mathbf{g} \mathbf{1}^k \mathbf{g}) = \mathbf{e} \quad \text{(3.2.224)} \]

Let $G_1$ and $G_2$ be two isogroups with respective isounits $\mathbf{I}_1$ and $\mathbf{I}_2$. The direct
isogroup $G = G_1 \circ G_2$ is the isogroup of all ordered pairs

\[ (\mathbf{I}_1, \mathbf{I}_2) \] and gives an explicit example. The conjecture was subsequently

proved by the late mathematician Gr. Tsagas [42] in 1996 for all simple Lie

groups of type A, B, C, D. The preceding departure of Prof. Tsagas while

working on the problems prevented him to complete the proof of the conjecture

for the case of all exceptional Lie groups. As a result, the proof of the indicated

conjecture remain incomplete at this writing.

For the unification here considered it is important to eliminate the restriction

that the isometries are necessarily positive definite, while preserving all other char-

acteristics, such as newri singularity and Hemiessity. As a result, a its simple

possible form, the isometries can be diagnosted into the form whose elements
can be either positive or negative,

\[ I = \text{Diag}(s_1, s_2, \ldots, s_k) \quad s_k \in R, s_k \neq 0, k = 1, 2, \ldots, N \]

The example provided in the original proposal [4], subsequently studied in
detail in Ref. [8], consisted in the classification of all possible simple Lie algebras
dimension $\mathfrak{d}$. In this case, Cartan's classification produces two non-isomorphic:
life algebras, the compact realtional algebras (in three dimension $SO(3)$ and

the noncompact algebra $SO(2)$).

The distinction between compact and noncompact algebras is lost under the

case of isopoints here considered. In fact, the classification of all possible, simple,

three-dimensional Lie-Santilli isometries $\mathbf{I}$ for the case of diagonal isometries

is characterized by the isometric itself and can be written

\[ I = \text{Diag}(s_1, s_2, \ldots, s_k) \quad s_k \in R, s_k \neq 0, k = 1, 2, \ldots, N \quad \text{(3.2.22)} \]

(3.2.225)

Interested readers can then easily derive any additional needed isogonal prop-

erty.

3.2.13 Unification of All Simple Lie Algebras into

Lie-Santilli Isometries

The original proposal [4] of 1978 included the conjecture that all simple Lie

algebras of dimension $N$ can be unified into a single Lie-Santilli isometry of

the same dimension, and gave an explicit example. The conjecture was consequently

proved by the late mathematician G. Tsagas [42] in 1996 for all simple Lie

groups of type A, B, C, and D. The premature departure of Prof. Tsagas while

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acteristics, such as newri singularity and Hemiessity. As a result, a its simple

possible form, the isometries can be diagnosted into the form whose elements
can be either positive or negative,

\[ I = \text{Diag}(s_1, s_2, \ldots, s_d, s_2) \quad s_k \in R, s_k \neq 0, k = 1, 2, \ldots, N \]

The example provided in the original proposal [4], subsequently studied in
detail in Ref. [8], consisted in the classification of all possible simple Lie algebras
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case of isopoints here considered. In fact, the classification of all possible, simple,

three-dimensional Lie-Santilli isometries $\mathbf{I}$ for the case of diagonal isometries

is characterized by the isometric itself and can be written

\[ I = \text{Diag}(I_1, I_2, \ldots, I_d) \quad I_k \in SO(2) \quad \text{(3.2.225)} \]

Interested readers can then easily derive any additional needed isogonal prop-

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can be either positive or negative,

\[ I = \text{Diag}(s_1, s_2, \ldots, s_d, s_2) \quad s_k \in R, s_k \neq 0, k = 1, 2, \ldots, N \]

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three-dimensional Lie-Santilli isometries $\mathbf{I}$ for the case of diagonal isometries

is characterized by the isometric itself and can be written

\[ I = \text{Diag}(I_1, I_2, \ldots, I_d) \quad I_k \in SO(2) \quad \text{(3.2.225)} \]

Interested readers can then easily derive any additional needed isogonal prop-

erty.

3.2.13 Unification of All Simple Lie Algebras into

Lie-Santilli Isometries

The original proposal [4] of 1978 included the conjecture that all simple Lie

algebras of dimension $N$ can be unified into a single Lie-Santilli isometry of

the same dimension, and gave an explicit example. The conjecture was consequently

proved by the late mathematician G. Tsagas [42] in 1996 for all simple Lie

groups of type A, B, C, and D. The premature departure of Prof. Tsagas while

working on the problems prevented him to complete the proof of the conjecture

for the case of all exceptional Lie algebras. As a result, the proof of the indicated

conjecture remain incomplete at this writing.

For the unification here considered it is important to eliminate the restriction

that the isometries are necessarily positive definite, while preserving all other char-

acteristics, such as newri singularity and Hemiessity. As a result, a its simple

possible form, the isometries can be diagnosted into the form whose elements
can be either positive or negative,
3.3 CLASSICAL LIE-ISOTOPIC MECHANICS FOR

3.3.1 Introduction

One of the reasons for the majestic consistency of quantum mechanics is the existence of axiomatic consistency and invariant classical formulations. Classical Lagrangian and Hamiltonian mechanics, namely, the discipline based on the fractional analytic equations

\[ \frac{d}{dt} (L(t, r, v)) = 0. \]  

3.3.1.4 The Fundamental Theorem for Isosymmetries and Their Isoalgebras

The fundamental symmetries of the 20th-century physics deal with point-like abstractions of particles in vacuum under linear, local and potential interactions, and are the Galilean symmetry (3.1.3) for nonrelativistic treatment or the Poincaré symmetry for relativistic formulations.

A central objective of mathematics is the broadening of those fundamental symmetries to represent extended, nonpolarizable and deformable particles under linear and nonlinear, local and nonlocal and potential as well as nonmonotonic interactions.

In fact, as we shall see, all novel industrial applications of hadronic mechanics must be crucially dependent on the admission of the extended character of particles or of their wavepackets in conditions of deep mutual penetration.

The latter conditions imply new effects permitting basically new energies and fields that are completely absent for conventional spacetime and other symmetries. Alternatively and equivalently a central problem of hadronic mechanics is the construction in an explicit form of the symmetries of all possible nonsuperspace, but otherwise arbitrary deformations of conventional spacetime and internal isovectors.

All these problems and others are resolved by the following important:

THEOREM 3.3.1.1: Let G be an N-dimensional Lie symmetry group of a K-dimensional metric or pseudo-metric space S(x, m, F) over a field F,

\[ G: x' = \Lambda(x) \times x, \quad y' = \Lambda(y) \times y, \quad x, y \in \hat{S}. \]  

\[ (x' - y')^2 \times x' \times m \times k = (x' - y') \cdot x' \times m \times (x - y). \]  

\[ (x' - y')^2 \times x' \times m \times k = (x' - y') \cdot x' \times m \times (x - y). \]

\[ \Lambda^0(x) = m \times \Lambda(x). \]

Then, all infinitely possible isosymmetries \( G \) of G acting on the isospace \( S(x, m, F) \)

\[ G: x' = \Lambda(x) \times x, \quad y' = \Lambda(y) \times y, \quad x, y \in \hat{S}. \]  

\[ (x' - y')^2 \times x' \times m \times k = (x' - y') \cdot x' \times m \times (x - y). \]

\[ \Lambda^0(x) = m \times \Lambda(x). \]

\[ k = 1, 2, 3, \ldots, N. \]

with a unique and unambiguous map into operator forms.

Following the original proposal [5] of 1976 to build hadronic mechanics, this author did consider the new discipline sufficiently mature for experimental verifications and industrial applications until the new discipline had equally consistent and invariant classical foundations with an equally unique and unambiguous map into operator formulations.

Interim: the operator foundations of hadronic mechanics were sufficiently identified in the original proposal [5], as we shall see in the next section. However, the identification of the classical counterpart turned out to be a rather complex task that required decades of research.

The objective, fully identified in 1978, was the construction of a covering of classical Lagrangian and Hamiltonian mechanics, namely, a covering of (3.3.11), admitting a unique and unambiguous range to the already known Lie-isotopic equations of hadronic mechanics.

The mandatory starting point was the consideration of the true Lagrange and Hamilton equations, those with external terms

\[ \frac{d}{dt} (L(t, r, v)) = \frac{\partial H(t, r, v)}{\partial \dot{r}} - \frac{\partial H(t, r, v)}{\partial \dot{v}} = 0. \]  

3.3.2 Insufficiencies of Analytic Equations with External Terms

It was indicated by Santilli [4] also in 1978 (see the review in Chapter 1 for more details) that the true analytic equations cannot be used for the construction of a consistent covering of conventional analytic equations because the new algebraic
brackets of the time evolution of a generic quantity $A(r,v)$ in phase space $\frac{\partial A}{\partial t} = [A, H] = [A, H] + \left(\frac{\partial A}{\partial r} \times \frac{\partial H}{\partial v} - \frac{\partial A}{\partial v} \times \frac{\partial H}{\partial r}\right) = \left(\frac{\partial A}{\partial r} \times \frac{\partial H}{\partial v} - \frac{\partial A}{\partial v} \times \frac{\partial H}{\partial r}\right) + \left(\frac{\partial A}{\partial r} \times \frac{\partial H}{\partial v} - \frac{\partial A}{\partial v} \times \frac{\partial H}{\partial r}\right)\left(\frac{\partial A}{\partial r} \times \frac{\partial H}{\partial v} - \frac{\partial A}{\partial v} \times \frac{\partial H}{\partial r}\right)$

violate the right distributive and scalar laws, Eqs. (3.2.5) and (3.2.6). Consequently, the true analytic equations in their original formulation are "all" impossible algebra, let alone possible Lie algebras. No axiomatically consistent covering can then be built under those premises.\(^{21}\)

The above insufficiency essentially established the need of rewriting the true analytic equations into a form admitting a consistent algebra in the brackets of the time evolution laws and, in addition, achieve the same invariance possessed by the truncated analytic equations.

Even though its main lines were fully identified in 1976, the achievement of the new covering mechanics resulted to require a rather long and laborious scientific journey.

This section is intended to outline the final formulation of the classical mechanics underlying hadronic mechanics in order to distinguish it from the numerous attempts that were published with the passing of time. As a brief guide to the literature, the reader should be aware that the true analytic equations (3.2.2) are generally set for open nonconservative systems. These systems require the broader Lie-admissible branch of hadronic mechanics that will be studied in the next chapter.

Therefore, the reader should be aware that several advances in Lie-isotopies have been obtained and can be originally identified as particular cases of the broader Lie-admissible theory.

The chapter is dedicated to the study of classical and operator closed-related systems verifying conventional total conservation laws while having the linear and nonlinear, local and nonlocal as well as potential and nonpotential internal forces.

The verification of conventional total conservation laws requires classical brackets that, firstly, verify the right and left distributive and scalar laws (as a condition to characterize an algebra), and, secondly, the brackets are necessarily antisymmetric.

The brackets of conventional Hamiltonian mechanics are Lie. Therefore, a necessary condition to build a true covering of Hamiltonian mechanics is the search of brackets that are of the broader Lie-isotopic type. As a matter of fact, this feature, fully identified in 1979 [4,5], was the very motivation for the construction of the Lie theory reviewed in Section 3.12.2.

In summary, the construction of a covering of the conventional Hamiltonian mechanics as the classical foundations of the Lie-isotopic branch of hadronic mechanics must be restricted to a reformulation of the true analytic equations (3.2.2) in such a way that the underlying brackets are Lie-isotopic, and the resulting mechanics is invariant.

### 3.3.3 Insufficiencies of Birkhoffian Mechanics

Santilli dedicated the second volume of Foundations of Theoretical Mechanics published by Springer-Verlag [2] in 1982 to the construction of a covering of classical Hamiltonian mechanics along the above indicated requirement. The resulting new mechanics was released under the name of Birkhoffian mechanics to honor G. D. Birkhoff who first discovered the underlying analytic equations in 1927.\(^{22}\)

Conventional Hamiltonian mechanics is based on the canonical action principle

$$\delta A = \int \left(\frac{\partial A}{\partial r} \times \frac{\partial H}{\partial v} - \frac{\partial A}{\partial v} \times \frac{\partial H}{\partial r}\right) = 0$$

and, via the use of the unified notation

$$\delta A = \int \left(\frac{\partial A}{\partial r} \times \frac{\partial H}{\partial v} - \frac{\partial A}{\partial v} \times \frac{\partial H}{\partial r}\right) = 0$$

from the conventional Hamilton's equations (3.3.1b) acquire the unified form

$$\delta A = \int \left(\frac{\partial A}{\partial r} \times \frac{\partial H}{\partial v} - \frac{\partial A}{\partial v} \times \frac{\partial H}{\partial r}\right) = 0$$

This feature, fully identified in 1979 [4,5], was the very motivation for the construction of the Lie theory reviewed in Section 3.12.2.

Moreover, Birkhoffian mechanics was proved in Ref. [2] to be "directly universal," that is, capable of representing "all" possible (sufficiently smooth and regular) Newtonian systems directly in the "frame of the observed" without any need for the transformation theory.

Therefore, at the time of releasing monograph [2] in 1982, the Birkhoffian mechanics appeared to have all the necessary pre-requirements to be the classical foundation of hadronic mechanics.

Unfortunately, subsequent studies established that Birkhoffian mechanics cannot be used for consistent physical applications because it is afflicted by the catastrophic inconsistencies studied in Section 1.4.1, with particular reference to the lack of invariance, namely, the inability to predict the same numbers for the same physical conditions at different times owing to the noncanonical character of the time-evolution.

Moreover, canonical action (3.3.4) is independent from the moments, $A^* = A^*(r)$, while this is not the case for the Pfaffian action (3.5.11), for which we have $A^* = A^*(r, \mu)$. Consequently, any map into an operator form implies "wave-functions" dependent on both coordinates and momenta, $y(r, \mu)$. Therefore, the operator image of Birkhoffian mechanics is beyond our current knowledge, and its study is deferred to future generations.

The above problems requested the resumption of the search for the consistent classical counterpart of hadronic mechanics from its beginning.

Numerous additional generalized classical mechanics were identified but they still missed the achievement of the crucial invariance (for brevity, see monographs [15,16] of 1994 and the first edition of monograph [6.7] of 1993).

By looking in retrospect, the origin of all the above difficulties resulted to be where one would expect them the least, in the use of the ordinary differential calculus.

Following the discovery in 1995 (see the second edition of monographs [6.7] and Ref. [10] of the nondifferential calculus, the identification of the final, axiomatically consistent and invariant form of the classical foundations of hadronic mechanics emerged quite rapidly.

### 3.3.4 Newton-Santilli Isomechanics for Matter and its Isodual for Antimatter

The fundamental character of Newtonian Mechanics for all scientific inquiries is due to the preservation at all subsequent levels of treatment (such as Hamiltonian mechanics, Galileo's relativity, special relativity, quantum mechanics, quantum chemistry, quantum field theory, etc.) of its main structural features, such as:

1) The underlying local-differential Euclidean topology.
2) The ordinary differential calculus; and
3) The consequential point-like approximation of particles.

---

\(^{21}\) Interested readers should consult, for brevity, the historical notes of Ref. [6].

\(^{22}\) The equations are called "analytic" in the sense of being derivable from a variational principle.
Nevertheless, Newton’s equations have well known notable limitations to maintain such a fundamental character for the entirety of scientific knowledge without due generalization for so many centuries.

As indicated in Chapter 1, the point-like approximation is indeed valid for very large mutual distances among particles compared to their size, as occurring for planetary and atomic systems (interior dynamical systems). However, the same approximation is inaccurate for systems of particles at short mutual distances, as occurring for the structure of planets, galaxies, nuclei and stars (interior dynamical systems).

Also, dimensionless particles cannot experience any contact or resistive interactions. Consequently, dissipative or, more generally, nonconservative forces used for centuries in Newtonian mechanics are a mere approximation being generally achieved via power series expansion in the velocities. It should be finally recalled on historical grounds that Newton had to construct the differential calculus as a pre-requisite for the formulation of his celebrated equations.

No genuine broadening of the disciplines of the 19th century is possible without a consistent structural generalization of their foundations, Newton’s equations in Newtonian mechanics.

Santilli’s isoaccentic mathematics has been constructed to permit the first unreasonably consistent structural generalization of Newton’s equations in Newtonian mechanics since Newton’s time, for the representation of extended, nonhomogeneous and deformable particles under linear and bi-linear, local and nonlocal and potential as well as nonpotential interactions as occurring in the physical reality of interior dynamical systems.

By following Newton’s teaching, the author has dedicated primary efforts to the approximation being generally achieved via power series expansion in the velocities.

For the above total unit can be factorized into the production of seven individual units for time and the two sets of individual Euclidean axes x, y, z with corresponding factorization of the fields

\[ \mathbf{\hat{R}}_x = \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x = \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x, \]

(3.3.16)

with total unit

\[ \mathbf{\hat{R}}_x = \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x. \]

(3.3.17)

The above total unit can be factorized into the production of seven individual units for time and the two sets of individual Euclidean axes x, y, z with corresponding factorization of the fields

\[ \mathbf{\hat{R}}_x = \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x = \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x, \]

(3.3.16)

with total unit

\[ \mathbf{\hat{R}}_x = \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x. \]

(3.3.17)

The Newton-Santilli isoonematics is then formulated on the 7-dimensional iso-space

\[ \hat{S}_{\text{tot}} = I_{\hat{R}}(I_{\hat{R}}) \times I_{\hat{R}}(I_{\hat{R}}) \times I_{\hat{R}}(I_{\hat{R}}). \]

(3.3.20)

with isometrics

\[ \hat{I}_x = T_x \times \hat{I}_x \times T_x \times \hat{I}_x, \]

(3.3.21)

over the Kneser product of isoids

\[ \hat{R}_{\text{tot}} = \hat{R}_x \times \hat{R}_x \times \hat{R}_x. \]

(3.3.22)

with total isoinvariant

\[ \hat{I}_{\text{tot}} = \hat{I}_x \times \hat{I}_x \times \hat{I}_x = \]

\[ \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x \times \mathbf{\hat{R}}_x. \]

(3.3.23)

Consequently, the isoinvariant can also be factorized into the product of seven distinct isomorphisms, with related product of seven distinct isoids

\[ \hat{I}_{\text{tot}} = \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x. \]

(3.3.24)

\[ \hat{R}_{\text{tot}} = \hat{R}_x \times \hat{R}_x \times \hat{R}_x \times \hat{R}_x \times \hat{R}_x \times \hat{R}_x \times \hat{R}_x. \]

(3.3.25)

and consequential applicability of the fundamental Tousso-Santilli-Santilli-Falcón-Núñez topology (or TSSFN Topology) that allows, for the first time to the author’s best knowledge, a consistent representation of extended, nonhomogeneous and deformable shapes of particles in Newtonian mechanics, here represented via the semians

\[ \hat{I}_x = \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x. \]

(3.3.26)

The Newton-Santilli isoonematics [6.10] can be written

\[ m v_0 = \hat{I}_x \times \hat{I}_x \times \hat{I}_x. \]

(3.3.27)

The Newton-Santilli isoonematics [6.10] can be written

\[ m v_0 = \hat{I}_x \times \hat{I}_x \times \hat{I}_x. \]

(3.3.27)

namely, the equations are concerned in such a way to formally coincide with the conventional equations for selfadjoint forms when formalized on isospace over selfadjoint forms, while all nonpotential forces are represented by the isoovent or, equivalently, by the isoadditive calculus.

Such a conception is the only known permitting the representation of extended particles with contact interactions that is invariant, thus avoiding the catastrophic inconsistencies of Section 1.4.1 and, in addition, achieves closure, namely, the verification of all conventional total conservation laws.

An inspection of Eqs. (3.3.27) is sufficient to see that the Newton-Santilli isomorphism reconsiders density, locality and linearity on isospace over selfadjoint forms, as studied in Section 3.2.11. Note that this would not be the case if nonselfadjoint forces appear in the right-hand side of Eqs. (3.3.27) as in Eqs. (3.3.2). Note the truly crucial role of the isoadditive calculus for the above structural generalization of Newtonian mechanics (as well as of the subsequent mechanics), that justifies a posteriori its conservation.

The verification of conventional total conservation laws is established by a visual inspection of Eqs. (3.3.27) since their symmetry is the Galilean isomorphism [14.15] that is isometric to the conventional Galilean symmetry, only formulated on isospace over selfadjoint forms. By recalling that conservation laws then follow from the indicated invariance.

When projected in the conventional representation space \( S_{\text{abs}} \), Eqs. (3.3.27) can be explicitly written

\[ m v_0 = \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x. \]

(3.3.28)

\[ m v_0 = \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x \times \hat{I}_x. \]

(3.3.29)

and they admit a solution, since they constitute a system of \( 6N + 1 \) unknowns given by \( \hat{I}_x \) and the diagonal \( N \times N \)-dimensional matrices \( \hat{I}_x \) and \( \hat{I}_x \).
Note that for $T_2 = 1$ we recover from a dynamical viewpoint the condition $\dot{\lambda}_i = \lambda_i$, obtained in Section 3.2.4 and 3.2.10 on geometric grounds.

As a simple illustration among unlimited possibilities, we have the following equation of motion of an extended particle with the ellipsoidal shape experiencing a resistive force $F_{\text{res}}(\mathbf{r}, t) = -\mathbf{r}$ because moving within an inner medium.

$$m \ddot{\mathbf{r}} = \int \Gamma (r, r', t) \mathbf{e} \cdot \mathbf{e} \, dV \quad \text{or} \quad m \ddot{r} = -r,$$  

where the so-called integral character with respect to a kernel $\Gamma$ is emphasized. Interested readers can then construct the representation of any desired non-Hamiltonian Newtonian system (see also memoir [10] for other examples).

Recall also that the optimal control theory can only be constructed for the most important systems, those significant for optimization. We, therefore, have the following important:

The direct universality of the Newton-Santilli isoequations, namely, the reconstruction of canonicity on extended, and, therefore, of nonlocal type.

Note that the action principle has the important application via the use of extended, and, therefore, of nonlocal-integral systems, thanks also to the underlying TSSFN isoduality and the isotopic image of the canonical antiparticles.

3.3.5 Hamilton-Santilli Isomechanics for Matter and its Isodual for Antimatter

3.3.5A. Isoaction Principle and its Isodual.

The fundamental isoaction principle can be written in unified notation:

$$S_{\text{iso}} = \int_{t_1}^{t_2} L(\mathbf{r}, \dot{\mathbf{r}}, t) \, dt = \int_{t_1}^{t_2} \left( H + \frac{1}{2} \dot{\mathbf{r}} \cdot \mathbf{F} \right) \, dt,$$

where $H = H(\mathbf{r}, \mathbf{p})$ is the Hamiltonian, $\mathbf{p}$ is the isomomentum, $\mathbf{F}$ is the action force.

This fundamental isoaction principle for the classical treatment of matter in interior conditions can be written in the explicit form in the $\mathbf{r}$ and $\mathbf{p}$ coordinates:

$$S_{\text{iso}} = \int_{t_1}^{t_2} \left( \mathbf{p} \cdot \mathbf{F} - \frac{1}{2} \mathbf{p} \cdot \mathbf{F} \right) \, dt = 0,$$

where $\mathbf{F} = \mathbf{F}(\mathbf{r}, \mathbf{p})$ is the action force, $\mathbf{p}$ is the isomomentum.

The following new feature now appears. The isospace, or, more technically, the isocotangent bundle of the isosymplectic geometry in local isochart $(\mathbf{r}, \mathbf{p})$ requires that the isounits of the variables $\mathbf{r}$ and $\mathbf{p}$ are inverse of each other (Section 3.2.3 and 3.2.10).

$$L = \int_{t_1}^{t_2} \left( \mathbf{p} \cdot \mathbf{F} - \frac{1}{2} \mathbf{p} \cdot \mathbf{F} \right) \, dt > 0.$$

Consequently, by ignoring hereon for notational simplicity the indices for the N particles, the total isounit of the isospace can be written:

$$L = \int_{t_1}^{t_2} \left( \mathbf{p} \cdot \mathbf{F} - \frac{1}{2} \mathbf{p} \cdot \mathbf{F} \right) \, dt > 0,$$

where $\mathbf{F} = \mathbf{F}(\mathbf{r}, \mathbf{p})$ is the action force, $\mathbf{p}$ is the isomomentum.

By using the unified notation

$$L = \int_{t_1}^{t_2} \left( \mathbf{p} \cdot \mathbf{F} - \frac{1}{2} \mathbf{p} \cdot \mathbf{F} \right) \, dt > 0,$$

where $\mathbf{F} = \mathbf{F}(\mathbf{r}, \mathbf{p})$ is the action force, $\mathbf{p}$ is the isomomentum.

The following new feature now appears. The isospace, or, more technically, the isocotangent bundle of the isosymplectic geometry in local isochart $(\mathbf{r}, \mathbf{p})$ requires that the isounits of the variables $\mathbf{r}$ and $\mathbf{p}$ are inverse of each other (Section 3.2.3 and 3.2.10).

$$L = \int_{t_1}^{t_2} \left( \mathbf{p} \cdot \mathbf{F} - \frac{1}{2} \mathbf{p} \cdot \mathbf{F} \right) \, dt > 0.$$
It is now important to show that Eqs. (3.3.45) provide an identical reformulation of the true analytic equations (3.1.2). For this purpose, we assume the simple case in which antisym coincide with the conventional time, that is, $I = t$, $L = 1$ and we write isoequations (3.3.45) in the explicit form

$$\begin{align*}
\rho \circ \partial \frac{\partial H}{\partial \rho} & = \left( \frac{\partial H}{\partial \rho} \right)_{\rho = 0} - \left( \frac{\partial H}{\partial \rho} \right)_{\rho = \infty} = \frac{d^h}{dh} \left( \frac{\partial H}{\partial \rho} \right)_{\rho = 0} - \left( \frac{\partial H}{\partial \rho} \right)_{\rho = \infty} \\
\rho \circ \partial \frac{\partial H}{\partial \rho} & = \left( \frac{\partial H}{\partial \rho} \right)_{\rho = 0} - \left( \frac{\partial H}{\partial \rho} \right)_{\rho = \infty} = \frac{d^h}{dh} \left( \frac{\partial H}{\partial \rho} \right)_{\rho = 0} - \left( \frac{\partial H}{\partial \rho} \right)_{\rho = \infty}
\end{align*}
$$

(3.3.48)

It is easy to see that Eqs. (3.3.48) coincide with the true analytic equations (3.1.2) under the trivial algebraic identification

$$L = diag\left\{ -I, \ldots, -I \right\};$$

(3.3.49)

As one can see, the main mechanism of Eqs. (3.3.45) is that of transferring the external terms $F = \mathbb{F}^{\sum /}$ into an explicit realization of the moment $L$. As a consequence, reformulation (3.3.45) constitutes direct evidence on the capability to represent non-Hamiltonian forces and effects with a generalization of the unit of the theory.

Note in particular that the external terms are embedded in the coherences. However, when written down explicitly, Eqs. (3.3.2) and (3.3.45) coincide. Note also that $L_i$ as in rule (3.3.49) is fully symmetric, thus acceptable as the isomorphism of isomechanics. Note also that all nonlocal and nonuniverse effects are embedded in $L$.

The reader should note the extreme simplicity in the construction of a representation of given non-Hamiltonian equations of motion, due to the algebraic character of identifications (3.3.49).

Recall that Hamilton’s equations with external terms are not derivable from a variational principle of the operator counterpart of Eqs. (3.3.2) throughout the 20th century. In fact, all differences between $L$ and $I$, and $x$, and $\partial$, etc. disappear at the abstract level. This proves the achievement of a central objective of isomechanics, the property that the analytic equations with external terms can indeed be identically rewritten in a form equivalent to the analytic equations without external terms, provided, however, that the reformulation occurs via the broader isomechanics.

The isodual Hamilton-Santilli isoequations for the classical treatment of antimatter, also identified soon after the discovery of the isodifferential calculus, are given by

$$\frac{d}{dt} \mathbb{H} = \mathbb{F}$$

(3.3.50)

$$\mathbb{H} = \mathbb{H}(x)$$

(3.3.51)

$$\mathbb{H} = \mathbb{H}(x)$$

(3.3.52)

and they verify the left and right distributive and scalar laws, thus characterizing a consistent algebra. Moreover, that algebra results to be Lie-isotopic, for which reasons the above brackets are known as the Lie-Santilli isoequations.
3.3.6 Simple Construction of Classical Isomechanics

The above classical isomechanics can be constructed via a simple method which does not need any advanced mathematics, yet it is sufficient and effective for practical applications.

In fact, the Hamilton-Santilli isomechanics can be constructed via the systematic application of the following noncanonical transformations to all quantities and operations of the conventional Hamiltonian mechanics

\[ U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

\[ U \times U' = I \]

\[ I_2 = I - \frac{F}{\partial H/\partial p} - I - \frac{F}{\partial H/\partial p} \] (3.164b)

The success of the construction depends on the application of the above noncanonical transform to the totality of Hamiltonian mechanics, with no exceptions. We have in this way the lifting of the six-dimensional unit of the conventional phase space into the isounit $I_2 - I_0 = I \times I_0$.

\[ n \rightarrow n = U \times n \times U' = n \times U \times I_0 = n \times I_0 \] (3.165)

3.4 OPERATOR LIE-ISOTOPIC MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

3.4.1 Introduction

We are finally equipped to present the foundations of the Lie-isotopic branch of nonholonomic mechanics for matter and its isodual for antimatter, more simply referred to as operator isomechanics, and its isodual for antimatter referred to as isovel operator isomechanics. The new mechanics will then be used in several sections for various developments, experimental verifications and industrial applications. The extension of the results of this section to relativistic operator isomechanics is elementary and will be done in the following sections whenever needed for specific applications. The case of operator isomechanics with a Lie-admissible rather than the Lie-isotopic structure, will be studied in the next chapter.

We know a section 3.2 is necessary for a technical understanding of operator isomechanics. For the mathematically non-inclined readers, we present in Section 3.4.3 a very elementary construction of operator isomechanics via homo-

\[ \frac{\partial x}{\partial \psi} = x \times \frac{\partial x}{\partial \psi} \] (3.167a)

\[ \frac{\partial y}{\partial \psi} = y \times \frac{\partial y}{\partial \psi} \]

\[ A' = U \times A \times U' \]

\[ I_{2n} \times x \times I_{2n} = \omega_x \times I_{2n} \]

\[ I_{2n} \times y \times I_{2n} = \omega_y \times I_{2n} \] (3.374)

Consequently, we have the following important theorem

THEOREM 3.3.5 (8,19): Following factorization of the isounit, isomechanical transformations are canonical.

The desired invariance of the Hamilton-Santilli isomechanics then follows. It is an instructive exercise for the reader interested in learning isomechanics to verify that all catastrophic mathematical and physical inconsistencies of non-canonical theories pointed out in Chapter 1 (see Section 1.4.1 in particular) are indeed resolved by isomechanics as presented in this section.
The use of Hamilton-Jacobi-Santilli isometrics (3.3.40) yields the following operator equations (here written for the simpler case in which $T$ has no dependence on $r$, but admits a dependence on velocities and higher derivatives)

$$-\dot{A}^\# + H^\# \tau + \mathcal{E}(\tau) = H + T \tau \mathcal{E}(\tau) = H + T \tau |\mathcal{E}(\tau)|,$$

$$\mathcal{F} |\mathcal{E}(\tau)| = \mathcal{F} |\mathcal{E}(\tau)|,$$  \hspace{1cm} (3.4.4a)

$$\mathcal{F} |\mathcal{E}(\tau)| = \mathcal{F} |\mathcal{E}(\tau)|,$$  \hspace{1cm} (3.4.4b)

that constitutes the fundamental equations of operator isomechanics, as we shall see in the next section.

As it is well known, Planck’s constant $\hbar$ is the basic unit of quantum mechanics. By comparing Eqs. (3.4.4a) and (3.4.4b) it is easy to see that $\hbar$ is the basic unit of operator isomechanics. Recall also that the isometries are defined at short distances as in Eqs. (3.1.40). We therefore have the following important

**POSTULATE 3.4.1 [5]:** In the transition from quantum mechanics to operator isomechanics Planck’s unit $\hbar$ is replaced by the interdependential unit $\hbar$ under the condition of removing the former at sufficiently large mutual distances,

$$\lim_{r \to \infty} \hbar r = 1.$$  \hspace{1cm} (3.4.5)

Consequently, the conditions of deep mutual penetration of the wave-packets and/or charge distributions of particles as studied by operator isomechanics there is the superposition of quantized and continuous exchanges of energy.

### 3.4.3 Isobilateral Spaces and their Isoduals

As it is well known, the Hilbert space $\mathcal{H}$ is used in quantum mechanics is expressed in terms of states $|\psi\rangle$, with normalization

$$|\langle \psi |\psi \rangle| = 1,$$  \hspace{1cm} (3.4.6)

and inner product

$$|\langle \psi |\psi \rangle| = \int \phi^\#(\tau) \times \phi(\tau),$$  \hspace{1cm} (3.4.7)

defined over the field of complex numbers $\mathcal{C} = \mathbb{C}, (\mathcal{A})$. The lifting $\mathcal{C}(\mathcal{A}) = \mathcal{C}(\mathcal{A})$, requires a compatible lifting of $\bar{H}$ into the isobilateral space $\bar{\mathcal{H}}$ as isoket $|\psi\rangle$, isometric product and isomorphuanization,

$$|\langle \psi |\psi \rangle| = \int \bar{\phi}^\#(\tau) \times \bar{\phi}(\tau) \times I \in \bar{\mathcal{C}},$$  \hspace{1cm} (3.4.8a)

$$|\langle \psi |\psi \rangle| = 1.$$  \hspace{1cm} (3.4.8b)

### 3.4.4 Structure of Operator Isomechanics and its Isodual

The structure of operator isomechanics is essentially given by the following main steps [47].

1) The description of closed-isolated systems is done via two quantities, the Hamiltonian representing all action-at-a-distance potential interactions, plus the isometric representing all non linear, nonlocal and non-Hamiltonian effects,

$$R(t, r, p) = \frac{p^2}{2m} + V(r),$$  \hspace{1cm} (3.4.16a)

$$I = I(r, p, \nabla, \ldots).$$  \hspace{1cm} (3.4.16b)

The explicit form of the Hamiltonian is that conventionally used in quantum mechanics although written on isomes as overfield, $\bar{H} = \bar{p}^2/2m + U(r)$.

$$\bar{H} = \bar{p}^2/2m + U(r).$$  \hspace{1cm} (3.4.17)

A generic expression of the isometric for the representation of two spinning particles with point-like change (such as the electron) in conditions of deep penetration of their wave-packets (as occurring in chemical valence bonds and many other cases) is given by

$$I = \exp \left[ \int \phi^\#(\tau) \times \bar{\phi}(\tau) \times r \right],$$  \hspace{1cm} (3.4.18)

where the nonlinearity is expressed by $G(\tau, \phi^\#(\tau))$ and the nonlocality is expressed by the volume integral of the deep wave-overlapplings $\int d\phi \phi^\#(\tau) \times \bar{\phi}(\tau)$. All isometries will be restricted by the condition of having positive-definite (thus everywhere invertible) as well as of recovering the trivial unit of quantum mechanics for sufficiently big mutual distances $r$,

$$\lim_{r \to \infty} \int d\phi \phi^\#(\tau) \times \bar{\phi}(\tau) = 0.$$  \hspace{1cm} (3.4.19)

2) The lifting of the multiplicative unit $I = 1 \to \bar{I} = 1/T \to \bar{T}$ requires the reconstruction of the entire formalism of quantum mechanics into such a form to

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*RuGGero Maria Santilli* first introduced by Myung and Santilli in 1982 [40] (see also monographs [8,7] for a comprehensive study).

It is easy to see that the isometric product is still inner (because $\bar{T} \geq 0$). Thus, $\bar{H}$ is still Hermitian and the lifting $\bar{N} = \bar{H}$ is an isometry. Also, it is possible to prove that iso-Hermitean associates with conventional Hermitean,

$$|\langle \psi |\psi \rangle| = |\langle \psi |\psi \rangle|,$$

$$\bar{R} = \bar{H} = \bar{H}.$$  \hspace{1cm} (3.4.9)

As a result, all quantities that are observable for quantum mechanics remain real for hadronic mechanics.

For consistency, the conventional eigenvalue equation $H |\psi\rangle = E |\psi\rangle$ must also be lifted into the iso-eigenvalue form

$$\bar{H} |\psi\rangle = \bar{E} |\psi\rangle \to |\psi\rangle = \bar{E} |\psi\rangle.$$  \hspace{1cm} (3.4.10)

where, in one case see, the final results are ordinary numbers.

Note the necessity of the isoptic action $\bar{H} |\psi\rangle$, rather than $\bar{H} |\psi\rangle$. In fact, only the former allows $I$ as the correct unit.

$$\bar{H} |\psi\rangle = \bar{T} \times |\psi\rangle.$$  \hspace{1cm} (3.4.11)

It is possible to prove that the iso-eigenvalues of isoket operators are isoradial, i.e., they have the structure $\bar{E} = \bar{E} = I, \bar{E} \in \mathcal{R}(n, +, \ldots)$. As a result all real eigenvalues of quantum mechanics remain real for hadronic mechanics.

We also recall the notion of isomembrane operators as the isomembrane $\bar{U}$ over $\bar{C}$ satisfying the isolates

$$\bar{U}|\psi\rangle = \bar{U}|\psi\rangle \Rightarrow \bar{U}.$$  \hspace{1cm} (3.4.12)

where we have used the identity $\bar{U} = \bar{U}$.

Finally indicate the notion of iso-eigenvalue value of an isomembrane $\bar{R}$ on $\bar{H}$ over $\bar{C}$

$$\bar{B} = \bar{R}|\psi\rangle = \bar{R}|\psi\rangle.$$  \hspace{1cm} (3.4.13)

It is easy to see that the iso-eigenvalues of isoket operators coincide with the isoeigenvalues, as in the conventional case.

Note also that the isometric value of the instant is the instant, $\bar{I} = \bar{I}$.

(provided, of course, that one uses the isoket (otherwise $\bar{I} = I$).

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The isotopy of quantum mechanics studied in the next sections are based on the novel isometric property of the conventional Hilbert space (xxx), here expressed in terms of a non-null scalar $\lambda$ independent from the integration variables

$$|\langle \psi |\psi \rangle| = \lambda,$$  \hspace{1cm} (3.4.15)

Note that new isometrics (3.4.15) remained undestroyed throughout the 20-th century because they required the prior discovery of new variables, these with arbitrary units.

### 3.5.1 Structure of Operator Isomechanics and its Isodual

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The explicit form of the Hamiltonian is that conventionally used in quantum mechanics although written on isopes as overfield,

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where the nonlinearity is expressed by $G(\tau, \phi^\#(\tau))$ and the nonlocality is expressed by the volume integral of the deep wave-overlapplings $\int d\phi \phi^\#(\tau) \times \bar{\phi}(\tau)$. All isometries will be restricted by the condition of having positive-definite (thus everywhere invertible) as well as of recovering the trivial unit of quantum mechanics for sufficiently big mutual distances $r$,

$$\lim_{r \to \infty} \int d\phi \phi^\#(\tau) \times \bar{\phi}(\tau) = 0.$$  \hspace{1cm} (3.4.19)

2) The lifting of the multiplicative unit $I = 1 \to I = 1/T \to \bar{T}$ requires the reconstruction of the entire formalism of quantum mechanics into such a form to
of quantum formalism. Their isotopic reformulations then press the invariance of hadronic mechanics, namely, its capability of predicting the same numbers for the same conditions at different times.

Under the above outlined structure we have the following main features

I) Hadronic mechanics is a coverage of quantum mechanics, because the latter theory is admitted uniquely and unambiguously at the limit when the isomatrices recover the conventional unit, $T = T_0$

II) Said covering is further characterized by the fact that hadronic mechanics coincides with quantum mechanics everywhere except for (as we shall see, generally small) non-Hamiltonian corrections at short mutual distances of particles caused by deep mutual overlapping of the wavepackets and/or charge distributions of particles.

III) Said covering is finally characterized by the fact that the indicated non-Hamiltonian corrections are restricted to verify all abstract axioms of quantum mechanics, with consequential preservation of a basic laws for closed non-Hamiltonian systems as a whole, as we shall see shortly.

Note that composite hadronic systems, such as hadrons, nuclei, isomers, etc., are represented via the tensorial product of the above structures. This can be best done via the identification first of the total system, total wavefield, total isomatrices, etc.,

$$\hat{\rho}_{\text{total}} = \hat{\rho}_1 \otimes \hat{\rho}_2 \otimes \ldots \otimes \hat{\rho}_N = \hat{\rho}_0 \otimes \hat{\rho}_0 \otimes \ldots$$

(3.2.45)

Note also that some of the units, fields and Hilbert spaces in the above tensorial products can be conventional, namely, the composite structure may implicitly include local-potential long range interactions (e.g., those of Coulomb type), which require the necessary treatment via conventional quantum mechanics, and nonlocal-nonpotential short range interactions (e.g., those in deep wave-overlapping), which require the use of operator isomechanics.


### 3.4.5 Dynamical Equations of Operator Isomechanics and their Isoisodals

The formalisms of the preceding sections permit the identification of the following fundamental dynamical equations of the Lie-isotopic branch of hadronic mechanics, known under the name of iso-Bohrmeule equations or isosymmetry.

Santilli isomatrices that were identified in the original proposal of 1979 to build hadronic mechanics [5], can be presented in their finite and infinitesimal forms, as shown in the following (3.2.46)

$$\dot{\mathbf{r}}_{\text{iso}} = \mathbf{v}_{\text{iso}} = \mathbf{v}^I = \mathbf{v}(\mathbf{r}, \mathbf{p}, \mathbf{\dot{r}}, \mathbf{\dot{p}}, \ldots) = \mathbf{U}^I$$

(3.2.46a)

$$\dot{\mathbf{v}}_{\text{iso}} = \mathbf{a}_{\text{iso}} = \mathbf{a}^I = \mathbf{a}(\mathbf{r}, \mathbf{p}, \mathbf{\dot{r}}, \mathbf{\dot{p}}, \ldots) = \mathbf{U}^I$$

(3.2.46b)

with the corresponding fundamental hadronic isocommutation rules

$$\mathbf{[v^a, v^b]} = (\mathbf{a}^a \times \mathbf{a}^b), \quad \mathbf{a}^0 = \mathbf{a}^I$$

(3.2.47)

namely, it can represent in the fixed coordinates of the experimentor all infinitely possible closed-relativistic systems with linear and nonlinear, local and nonlocal, and potential as well as nonpotential internal forces verifying the conservation of the total energy.

A consistent formulation of the isolunar momentum (3.4.29) occupied identification for two decades, thus delaying the completion of the construction of hadronic mechanics, as well as its practical applications. The consistent and invariant form (3.4.29) with consequent isomomentum commutation rules were first identified by Santilli in the second edition of Vol. II of this series, Ref. [7] of 1995 and memoir [10], following the discovery of the isomomentum calculus.


### 3.4.6 Preservation of Quantum Physical Laws

As one can see, the fundamental assumption of isomomentum is the lifting of the basic unit of quantum mechanics, Planck’s constant $\hbar$, into a matrix $\mathbf{I}$ with nonlinear, inter-differential elements which also depend on the wavefunction and its derivatives.

$$\mathbf{I} = \mathbf{I}^0 \otimes \mathbf{I} = \mathbf{I} \otimes \mathbf{I} = \mathbf{I} = \mathbf{I} \otimes \mathbf{I}$$

(3.4.32)

It should be indicated that the above generalization is only internal in non-localized, closed isochronous because, when measured from the outside, the isomatrices and isocommutators of the second order Planck’s constant identically [46],

$$\dot{\mathbf{I}} = \mathbf{0} \times \dot{\mathbf{I}} = \mathbf{I} \otimes \dot{\mathbf{I}} = \dot{\mathbf{I}}$$

(3.4.33a)

$$\mathbf{I} \otimes \dot{\mathbf{I}} = \dot{\mathbf{I}} \otimes \mathbf{I} = \dot{\mathbf{I}} \otimes \mathbf{I}$$

(3.4.33b)

$$\dot{\mathbf{I}} = \mathbf{I} \otimes \dot{\mathbf{I}} = \mathbf{I} \otimes \mathbf{I} = \mathbf{I}$$

(3.4.33c)

Moreover, the isomomentum is the fundamental isometric of isomechanics, preserving all isometric of the conventional unit $\mathbf{I} = 0$, e.g.,

$$\mathbf{I} \otimes \mathbf{I} = \mathbf{I} = \mathbf{I} \otimes \mathbf{I} = \mathbf{I} \otimes \mathbf{I}$$

(3.4.34a)

$$\dot{\mathbf{I}} = \mathbf{I} \otimes \mathbf{I} = \mathbf{I} \otimes \mathbf{I}$$

(3.4.34b)

$$\dot{\mathbf{I}} = \mathbf{I} \otimes \mathbf{I} = \mathbf{I} \otimes \mathbf{I}$$

(3.4.34c)

Despite their generalized structure, Eqs. (3.4.36) and (3.4.38) preserve conventional quantum mechanical laws under nonlinear, nonlocal and nonpotential interactions [7].

To begin an outline, the preservation of Heisenberg’s uncertainties can be easily derived from isocommutation rules (3.4.27)

$$\Delta^\mathbf{I} \times \Delta^\mathbf{p}_I \geq \frac{1}{2} \left(\bar{\mathbf{M}}^\mathbf{I} \mathbf{p}_I^2 \right) = \frac{1}{2} \left(\bar{\mathbf{M}}^\mathbf{I} \mathbf{p}_I^2 \right)$$

(3.4.35)
A simple lifting of the conventional perturbation expansion then yields
\[
\hat{E}(\hat{k}) = \hat{E}_0 + \hat{k} \cdot \hat{\mathbf{p}} + \hat{k}^2 \hat{\mathbf{p}} + \hat{k}^4 \hat{\mathbf{p}}^2 + \cdots
\]
(3.4.48)
whose convergence can be easily reached via a suitable selection of the isotopic element, e.g., such that \( |\hat{T}| < 1 \).

As an example, for a positive-definite constant \( \hat{T} \leq 1 \), expression (3.4.46) becomes
\[
\hat{E}(\hat{k}) = \hat{E}_0 + \hat{k} \cdot \hat{\mathbf{p}} + \frac{\hat{\mathbf{p}}^2}{\hat{T}} \cdot \hat{\mathbf{p}} + \frac{\hat{\mathbf{p}}^4}{\hat{T}^2} \cdot \hat{\mathbf{p}}^2 + \cdots
\]
(3.4.49)
This shows that the original divergent coefficients \( 1, \hat{k}^2, \cdots \) are now turned into the manifestly convergent coefficients \( 1, \hat{T}, \hat{k}^2 \cdot \hat{T}, \cdots \), with \( \hat{T} > 1 \) and \( \hat{T} \equiv 1/k \), thus ensuring isoconvergence for a suitable selection of \( \hat{T} \) for each given \( \hat{k} \) and \( \hat{V} \).

The presentation of the superposition principle under nonlinear interactions occurs because of the reconstruction of linearity in nonlinearity over isoids, thus requiring the applicability of the theory to composite systems.

Recall in this latter respect that conventionally nonlinear models,
\[
H(\hat{x}, \hat{r}, \hat{p}, \cdots) \times |\hat{\psi}\rangle = \hat{E} \times |\hat{\psi}\rangle,
\]
(3.4.42)
violate the superposition principle and have other shortcomings (see Section 1.5). As such, they cannot be applied to the study of composite systems such as molecules. All these models can be identically reformulated in terms of the isotopic techniques via the embedding of all nonlinear terms in the isotopic element,
\[
H(\hat{x}, \hat{r}, \hat{p}, \cdots) \times |\hat{\psi}\rangle = \hat{R}(\hat{\sigma}, \hat{\gamma}) \times \hat{T}(\hat{\nu}, \cdots) \times |\hat{\psi}\rangle = \hat{E} \times |\hat{\psi}\rangle,
\]
(3.4.43)
by regarding the full validity of the superposition principle in isoids over isoids with consequent applicability to composite systems.

The preservation of causality follows from the one-dimensional isomultiplicative structure of the time evolution (3.2.28) which is isomorphic to the conventional one: the preservation of probability laws follows from the preservation of the axioms of the unit and its invariant decomposition as indicated earlier; the preservation of other quantum laws then follows.

The same results can also be seen from the fact that operator isomechanics coincides at the abstract level with quantum mechanics by conception and construction. As a result, hadronic and quantum versions are different realizations of the same abstract axioms and physical rules.

Note that the preservation of conventional quantum laws under nonlinear, nonlocal and nonmetrical interactions is crucially dependent on the capability of nonisomathematics to reconstruct linearity, locality and causality-units on isoids over isoids.

The preservation of conventional physical laws by the isotopic branch of hadronic mechanics was first identified by Santilli in report [47]. It should be indicated that the same quantum laws are not generally preserved by the broader

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**3.4.7 Isoperm Theory and its Isoanal**

We are now sufficiently equipped to illustrate the computational advantages in the use of isotopes.

THEOREM 3.4.7: Under sufficient continuity conditions, all perturbative and other series that are conventionally divergent (weakly convergent) can be turned into convergent (strongly convergent) forms via the use of isotopes with sufficiently small unit element (sufficiently large complex).

In summary, the studies on the construction of hadronic mechanics have indicated that the apparent origin of divergences (or slow convergences) in quantum mechanics and chemistry is their lack of representation of nonlinear, nonlocal, and nonmetrical effects because when the latter are represented via the ison, full convergence (much faster convergence) can be obtained.

As we shall see, all known applications of hadronic mechanics verify the crucial condition \( |\hat{T}| > 1 \) (or \( |\hat{T}| > 1 \), by permitting convergence of perturbative series. For instance, in the case of chemical bonds, hadronic chemistry allows computations at least one thousand times faster than those of quantum chemistry, with evident advantages, e.g., a drastic reduction of computer time (see Chapter 9). Essentially the same conclusions are expected for hadronic phenomenons, such as superconductivity.

The reader should meditate a moment on the evident possibility that hadronic mechanics offers realistic possibilities of constructing a convergent perturbative theory for strong interations. As a matter of fact, the divergences that have afflicted strong interations theories represented by noetherian theory have been replaced by the even more serious approximation of hadronic as points, with the consequential sole potential interactions and related divergences.

In fact, whenever hadrons are represented as they actually are in reality, explicitly and nonpotential effects because when the latter are represented via the ison, full convergence (much faster convergence) can be obtained.

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In fact, whenever hadrons are represented as they actually are in reality, explicitly and nonpotential effects because when the latter are represented via the ison, full convergence (much faster convergence) can be obtained.
As we shall see in the next chapters, the above property provides means for the explicit construction of the new model of isomechanics from the conventional model. The main requirement is that of identifying the nonclassicalism effects one desires to represent, which as such, are necessarily invisible. The resulting nonuniform transform is then assumed as the new fundamental space of the new isomechanics [46]

\[ U \rightarrow U' = I \oplus I. \]

under which transform we have the liftings of the quantum unit into the isomatrix, where

\[ I = \hat{I} = U \otimes I \otimes U^T. \]

numbers into isomommers,

\[ a \rightarrow ^{\hat{a}} = a \otimes u \otimes u^T \}

\[ = x \otimes (u \otimes u^T) \otimes \hat{a} = a \otimes \hat{u} \otimes u^T, \]

associated product \( A \otimes B \) into the isomorphic form with the correct isotope element.

\[ \hat{A} = (\hat{A} \otimes \hat{U} \otimes \hat{U}^T) \otimes \hat{B} = \hat{A} \otimes \hat{B} \]

Schrödinger’s equation into its isomorphed’s equations

\[ H \otimes I = \hat{E} \otimes I \]

Heisenberg’s equations into their isochromeshic generalization

\[ i \frac{d}{dt} \hat{A} \otimes \hat{B} = \hat{A} \otimes \hat{B} - \hat{B} \otimes \hat{A} = 0 \]

the Hilbert product into its isomiser form

\[ (\xi | \eta) \rightarrow U (\hat{\xi} | \eta) = U \cdot \hat{\eta} \cdot \hat{\xi} \]

canonical power series expansion into its isomorphic form

\[ A(k) = \hat{A}(k) + k \hat{A}(k) \hat{B}(k) + k^2 \hat{A}(k) \hat{B}(k) + \cdots \]

Thus, the entire extismatter content of the universe cannot be credibly treated vis-à-vis special and/or general relativity.

A widespread academic posture, studiously conceived for adapting nature to a preferred doctrine, is that irreversibility is a macroscopic event that “disappears” (sic) when systems are reduced to their elementary constituents. This widespread academic belief is necessary because special and general relativity are structurally reversible, namely, their mathematical and physical axioms, as well as all known Hamiltonians are invariant under time reversal. This posture is complemented with manipulations of scientific evidence, such as the presentation of the probability of the stability of the two nuclei into a third one, \( n_1 + n_2 \rightarrow n_3 \) while statistically suppressing the time reversal event that is simply unavoidable for a reversible theory, namely, the finite probability of the spontaneous decoupling \( n_3 \rightarrow n_1 + n_2 \) following the synthesis. The latter probability is suppressed evidently because it would prove the inconsistency of the assumed basic doctrine. 25

Unfortunately for mankind, the above academic postures are also used for all energy releasing processes despite the fact that they are irreversible. The vast majority of the research on energies releasing processes such as the “cold” and “hot” fusion, and the use of the vast majority of public funds are restricted to verify quantum mechanics and special relativity under the knowledge by experts that reversible theories cannot be exactly valid for irreversible processes. In any case, the “No reduction theorems” prevent the consistent reduction of an

\[ E(k) = E(\hat{E}) + k \hat{E} \times I + k \hat{E} + k^2 \hat{E} \times \hat{E}. \]

Schrödinger’s perturbation expansion into its isotropic covering (where the usual summation over states \( s \) is assumed)

\[ \rightarrow U \rightarrow \hat{U} \]

et cetera. All remaining aspects of operator isomechanics can then be derived accordingly, including the isocomponent, isosignalement, isotrajectories, isoequipotential functions and transforms, etc. The isolevel mechanics can then be constructed, via the new familiar isomorphic map.

Note that the above construction via a nonuniform transform is the correct operator image of the de-terminism of the classial isomathematical mechanics from the conventional form of nonmathematical transforms (Section 3.2.3).

The construction of hadronic mechanics via nonuniform transforms of quantum mechanics was first identified by Santilli in the original proposal [5], and then worked out in subsequent contributions [see [2] for the latest presentation].

Schrödinger’s perturbation expansion into its isotropic covering (where the usual summation over states \( s \) is assumed)

\[ E(k) = E(\hat{E}) + k \hat{E} \times I + k \hat{E} + k^2 \hat{E} \times \hat{E} = E(\hat{E}) + E(\hat{E}) + k \hat{E} \times I + \hat{E} \times \hat{E}. \]

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3.4.9 Invariance of Operator Isomechanics and of its Isodual

It is important to see that, in a way fully parallel to the classical case (Section 3.5.7), operator isomechanics is indeed invariant under the most general possible nonliner, isochromeshic nonuniform transforms, provided that, again, the isomatrix is treated vis-à-vis the isomathematics. In fact, any given nonuniform transform \( U \otimes U' \otimes I \) can always be decomposed into the form \( \hat{U} \otimes \hat{U} \otimes \hat{I} \)

under which nonuniform transforms on \( I \) over \( C \) are identically reformulated as isomathematics on the isoinherent space \( \hat{N} \) over the isomedical \( \hat{U} \)

\[ U \otimes \hat{U} \otimes \hat{I} = \hat{U} \cdot \hat{E} \cdot \hat{B} \]

The form-invariance of operator isomechanics under isomathematics was first studied by Santilli in memores [46].

The construction of hadronic mechanics under isomathematics was

3.5.1 Limitations of Special and General Relativities

Special and general relativity are basically presented in contemporary academia as providing formal descriptions of all indefinitely possible conditions existing in the universe. The scientific reality is basically different than the above academic posture. In Section 1.1 and Chapter 2, we have shown that special and general relativities cannot provide a consistent classical description of antiparticles because they admit no distinction between neutral matter and antimatter and, when used for charged particles, they lead to inconsistent quantum images consisting of particles (rather than charge conjugated antiparticles) with the wrong sign of the charge. Hence, the entire extismatter content of the universe cannot be credibly treated vis-à-vis special and/or general relativity.

A widespread academic posture, studiously conceived for adapting nature to a preferred doctrine, is that irreversibility is a macroscopic event that "disappears" (sic) when systems are reduced to their elementary constituents. This widespread academic belief is necessary because special and general relativities are structurally reversible, namely, their mathematical and physical axioms, as well as all known Hamiltonians are invariant under time reversal. This posture is complemented with manipulations of scientific evidence, such as the presentation of the probability of the stability of the two nuclei into a third one, \( n_1 + n_2 \rightarrow n_3 \) while statistically suppressing the time reversal event that is simply unavoidable for a reversible theory, namely, the finite probability of the spontaneous decoupling \( n_3 \rightarrow n_1 + n_2 \) following the synthesis. The latter probability is suppressed evidently because it would prove the inconsistency of the assumed basic doctrine. 25

Unfortunately for mankind, the above academic postures are also used for all energy releasing processes despite the fact that they are irreversible. The vast majority of the research on energies releasing processes such as the “cold” and “hot” fusion, and the use of the vast majority of public funds are restricted to verify quantum mechanics and special relativity under the knowledge by experts that reversible theories cannot be exactly valid for irreversible processes.

In any case, the “No reduction theorems” prevent the consistent reduction of an
invariant macroscopic event to an ideal ensemble of point-like abstractions of particles all in reversible conditions. Hence, special and general relativity are inapplicable for any and all irreversible processes existing in the universe.7

When restricting the arena applicability to those of the original conception (propagation of point particle and electromagnetic waves in vacuum), special relativity remains affected by still unresolved basic problems, such as the possible that the relativistic verifying one-way experiments on the propagation of light could be Galilean, rather than Lorentzian, the known incompleteness of special relativity with space conceived as a universal medium; and other unsolved aspects. Independently from that, we have shown in Section 1.3 that general relativity has no case of unambiguous formulation for numerous reasons. Such as curvature cannot possibly represent the free fall of a body along a straight rigid line; the "bending of light" is due to Newtonian gravitation (and if curvature is assumed one gets double the bendings experimentally measured); gravitation is a noncovariant theory, thus suffering of the Theorems of Catastrophic Inconsistencies of Section 1.3, etc.

In summary, on serious scientific grounds, and contrary to vastly popular political beliefs, special and general relativities have no uncontested arena of exact validity.

From putting to good use the advice of our colleagues, we have been able to identify the above open problems and initiated quantititative studies for their resolution. Our position in regard to special relativity is that the universal constancy of the speed of light is a local variable depending on the characteristics of the medium in which it propagates, while the speed of light is generally bigger than that in vacuum when propagating within media of very high density, such as the interior of stars and quasars, etc.

As we shall see, when the memory of the founders is honored in the above sense, the broadest possible realization of their axioms include gravitation and there is no need for general relativity as a separate theory. Thus, another basic understanding of this section is that we shall seek a unification of special and general relativity into one simple formulation based on the axioms of special relativity, known as Santilli isorelativity. Needless to say, such a unification required several decades of research since it required the construction of the needed new mathematics, the achievement of the unification of the Minkowskian and Riemannian geometries, and the achievement of a universal invariance for all possible spacetime elements prior to addressing the unification itself.

A further aspect important for the understanding of this section is that, by means isorelativity should be believed to be the final relativity of the universe because it is structurally reversible due to the Hermiticity of the isounit and isotopic elements 14

This creates the need for a yet broader relativity studied in the next chapter, and known under the name of Santilli generalized relativity, this time, based on isophysical theories of special relativity or isorelativity, namely, broadening requiring a necessary departure from the abstract axioms of special relativity into a form that is structurally reversible, in the sense of possessing mathematical and physical axioms that are irreversible under all possible reversible Lagrangians or Hamiltonians.

The resolution of the above indicated problems for antimatter is achieved by the isodelta maps of the studies of this section.

5.2 Minkowski-Santilli Isospaces and their Isoduals

As studied in Section 1.2, the "universal constancy of the speed of light" is a philosophical abstraction, particularly when professed by experts without the additional crucial words "in vacuum", because the constancy of the speed of light has been solely proved in vacuum while, in general, experimental evidence establishes that the speed of light is a local variable depending on the characteristics of the medium in which it propagates, with well known expression

\[ c = c_{0}/\sqrt{\gamma}, \]

which the familiar index of refraction \( n \) is a function of a variety of time \( t \), coordinates \( x, y, z \), density \( \rho \), temperature \( T \), curvature, etc., \( n = n(t, x, y, z, \rho, T, \gamma, \ldots) \). In particular, the speed of light is generally smaller than that in vacuum when propagating within special guides, or within media of very high density, such as the interior of stars and quasars, etc.

Academic claims of recovering the speed of light via photon scattering among the water molecules are afflicted by numerous counterarguments studied in Section 1.2, and the same holds for other aspects. Assume that via some unknown mechanism, special relativity is shown to represent consistently the propagation of light within physical media, such a representation would actuate the catastrophic inconsistencies of Section 1.3.

This is due to the fact that the translation from the speed of light in vacuum to that within physical media requires a noncovariant or nonsymmetric transformation. This point can be best illustrated by using the metric recently proposed by Minkowski, which can be written

\[ g = \text{Dom}(1, 1, 1, -c^2). \]

Then, the transition from \( c_0 \) to \( c_{0}/\sqrt{\gamma} \) in the metric can only be achieved via a noncovariant or nonsymmetric transformation.

\[ g = \text{Dom}(1, 1, 1, -c^2) \Rightarrow g = \text{Dom}(1, 1, 1, -c_{0}^2) + U \times y + y \times U^t, \]

\[ U \times U^t = \text{Dom}(1, 1, 1, 1/\gamma) \neq I. \]

An invariant resolution of the limitations of special relativity for closed and reversible systems of extended and deformable particles under Hamiltonian and non-Hamiltonian interactions has been provided by the lifting of special relativity into a new formulation known as Santilli isorelativity, where the prefix "iso" stands to indicate that relativity principles apply on isospace over isochronal, and the characterization of "special" or "gamma" is unimportant because, as shown below, isorelativity achieves a geometric unification of special and general relativities.

Isorelativity was first proposed by R. M. Santilli in Ref. [58] of 1983 via the first invariant formulation of an Minkowskian space and related iso-Lorentz symmetry. The studies were then continued in Ref. [59] of 1985 with the first isopotential of the rotational symmetry: Ref. [40] of 1991 with the first isopotential of the SU(2)-spin symmetry: Ref. [41] of 1991 with the first isopotential of the Poincaré symmetry: Ref. [34] of 1998 with the first isopotential of the SU(2)-spin symmetries: Bell's inequality and local realism: and Refs. [61,62] of 1992 on the first isopotential of the spinorial covering of the Poincaré symmetry.

The studies were then completed with memoir [26] of 1998 presenting a comprehensive formulation of the isorelativity principles, which unify the Minkowskian and Riemannian geometries, including its formulation via the mathematics of the Riemannian geometry (such iso-Christoffel's symbols, isovariant derivatives, etc.). The author then dedicated various monographs to the field through the years.

Numerous independent studies on Santilli isorelativity are available in the literature, one can inspect in this respect Refs. [42–43] and papers quoted therein; Aringazin's proof [63] of the direct universality of the Lorentz-Poincare-Santilli isoinvariance for all infinitely possible isospacetimes with signature \((e,+,+,\ldots)\); Mignani's exact representation [64] for the large d'Alambertian of the shifting skewness between quasars and galaxies when physically connected; the exact representation of the experimental data on the Bose-Einstein correlation by Santilli [67] and Cardone and Mignani [58]; the invariant and exact validity of the iso-Minkowskian geometry within the hyperdense medium in the interior of hadrons by Arozter [61], the first known exact representation of molecular features by Santilli and Shalitze [61,62], and numerous other contributions.

Evidently we cannot review isorelativity in the necessary details to avoid a prohibitive length. Nevertheless, to achieve minimal self-sufficiency of this presentation, it is important to outline at least its main structural lines (see monograph [55] for detailed studies). The central notion of isorelativity is the lifting of the basic unit of the Minkowski space and of the Poincaré symmetry, \( I = \text{Dom}(1, 1, 1, 1) \), into a 4 x 4-dimensional, nowhere-singular and positive-definite matrix \( I = \text{Dom}(1, 1, 1, 1) \), with an unrestricted functional dependence on local spacetime coordinates \( x, \) speeds, etc.

\[ \text{Diag}: n = n(t, x, y, z, \rho, T, \gamma, \ldots) \]

14 Needless to say, the above sketch is not the entire picture. Despite its Hermiticity, the isounit can depend on time as such that \( H(t, x, y, z, \rho, T, \gamma, \ldots) = H(t + \Delta t, x, y, z, \rho, T, \gamma, \ldots) \) and the same holds for other aspects, wherein the isounit is continuously verified, evolving field conservation laws when nuclei (because of the antisymmetry of the Lorentz-Poincare-Santilli isounit and isounit isomorphisms), yet it is the case for all atomic problems when considered isolated from the rest of the universe.

\[ \text{Diag}: \text{Dom}(1, 1, 1, c^2) \]

\[ \text{Diag}: \text{Dom}(1, 1, 1, 1/\gamma) \neq I. \]

\[ U \times U^t = \text{Dom}(1, 1, 1, 1/\gamma) \neq I. \]
accretions \( \phi \), frequencies \( \omega \), wavefunctions \( \psi \), their derivative \( \partial \phi \), and/or any other needed variables.

\[
L = \text{Diag}(\psi(1,1,1) \rightarrow \psi(x,y,z,\phi,\theta,\cdots) = \text{Diag}(\psi(1,1,1,\cdots,\psi,\theta,\cdots) = 0. \tag{3.5.6}
\]

Isotropy can then be constructed via the method of Section 3.4.6, namely, by assuming that the basic noncanonical or nonunitary transform coincides with the above isounit

\[
U \times U^\dagger = \text{Diag}(\eta_{\alpha\beta}, \eta_{\gamma\delta}, \eta_{\mu\nu}) \tag{3.5.7}
\]

and then subjecting the totality of quantities and their operation of special relativity to the above transform. This construction is, however, selected here only for simplicity in pragmatic applications, since the rigorous approach is the construction of isotropy from its abstract antisitic nature, a task we have to leave to interested readers for brevity (see the original derivations [7]).

It is due to the fact that the former approach evidently preserves the original eigenvalue spectra and does not allow the identification of anomalous eigenvalue changes from the second approach, such as those of the SU(3) and SU(4) isosymmetries [5].

Let \( \mathcal{M}(\eta, \theta, R) \) be the Minkowski space with local coordinates \( x = (\eta^2) \), metric \( \eta = \text{Diag}(1,1,1,1) \) and invariant

\[
x^2 = (\eta^2 + \eta^3 + \eta^4) \times I \in \mathcal{R}. \tag{3.5.8}
\]

The fundamental space of isotropy is the Mandelbrocki isospace [58] and related topology [10,22-25]. \( \mathcal{M}(\eta, \theta, R) \) characterized by the liftings

\[
\eta = \text{Diag}(I,1,1,1) \times U \times U^\dagger = \text{Diag}(\eta^4 + \eta^5 + \eta^6 + \eta^7 + \eta^8 + \eta^9) \times \mathcal{I}, \tag{3.5.9a}
\]

with consequent isotropy of the basic invariant

\[
x^2 = (\eta^2 + \eta^3 + \eta^4) \times I \in \mathcal{R}, \tag{3.5.9b}
\]

whose projection in conventional spacetime can be written

\[
x^2 = (\eta^2 + \eta^3 + \eta^4) \times \mathcal{I} \in \mathcal{R}. \tag{3.5.9c}
\]

THEOREM 3.5.1: The Mandelbrocki isospaces are directly universal, in the sense of admitting as particular cases all possible spaces with the same signature \(+ + + + = 0 \), such as the Mandelbrocki, Riemannian, Euclidean and other spaces (universally), directly in terms of the isometry within fixed local variables (direct universality).

Thus, the correct formulation of the Mandelbrocki isospaces requires the isometry of all tools of the Riemannian geometry, such as the isometries of the sphere, local coordinates, etc. (see for brevity Ref. [35]).

Despite that, one should keep in mind that, in view of the positive definiteness of the isounit [54,79], the Mandelbrocki isospaces are not of the abstract level with the conventional Riemannian geometry, thus losing a well preserved sphere (because of the basic mechanism of deforming the metric \( g \) by the amount \( \eta_{M}^{\dagger} \), while deforming the basic unit of the inverse amount \( I = I^\dagger \)).

The geometric unification of the Mandelrohnian and Riemannian geometries achieved by the Mandelbrocki isospaces constitutes the evident geometric foundation for the unification of special and general relativity studied below.

Therefore, the above theorems in Ref. [58] are ignored in these studies because they are formulated via traditional mathematics and, consequently, they all suffer of the catastrophic inconsistencies of Theorem 3.5.1. By comparison, isospaces are formulated via isometrics and, therefore, they resolve the inconsistencies of Theorem 3.5.1, as shown in Section 3.5.9. This illustrates again the necessity of lifting the basic unit and related field jointly with all remaining, conventional mathematical methods.

3.5.3 Poincarè-Santilli Isospaceness and its Isovadal

Let P(3,1) be the conventional Poincarè symmetry with the well known ten generators, \( J_{\alpha\beta} \), and related commutation rules herein assumed to be known.

The second basic tool of isospaceness is the Poincarè-Santilli isospaceness \( P(3,1) \) studied in detail in monograph [55] that can be constructed via the isothermic

\[
[x^2 = (\eta^2 + \eta^3 + \eta^4) \times \mathcal{I} \in \mathcal{R}. \tag{3.5.10}
\]

LEMMA 3.5.1: The rotational symmetry remains exact for all possible signature-preserving (- + + + ) deformations of the sphere.

The rotational symmetry was believed to be broken for ellipsoidal and other deformations of the sphere merely due to insufficient mathematics for the case considered because, when the appropriate mathematics is used, the rotational symmetry turns to be exact, and the same holds for virtually all "broken" symmetries.

The above reconstruction of the exact rotational symmetry can be geometrically visualized by the fact that all possible signature-preserving deformations of the sphere are perfect spheres in isospace called isosphere.

This is due to the fact that ellipsoidal deformations of the semiaxes of the perfect sphere are compensated on isospaces over isofields by the isostrophy of all tools of the Riemannian geometry, thus having a null isocurvature in the sense of admitting as particular cases all possible spaces with the same signature \(+ + + + = 0 \).

Theorem 3.5.1 states that, for isomers in the (3, 2)-plane, are given by

\[
x^2 = x^2 \times \cos \theta \times (g_{11} + g_{22})^{\frac{1}{2}} - x^2 \times \sin \theta \times (g_{11} + g_{22})^{\frac{1}{2}}, \tag{3.5.1a}
\]

\[
x^2 = x^2 \times \cos \theta \times (g_{11} + g_{22})^{\frac{1}{2}} + x^2 \times \sin \theta \times (g_{11} + g_{22})^{\frac{1}{2}}, \tag{3.5.1b}
\]

For the general expression in three dimensions interested reader can inspect Ref. [7] for brevity.

Note that, since \( O(3) \) is isomorphic to \( O(3) \), Ref. [39] proved, contrary to a popular belief throughout the 20th century, that

LEMMA 3.5.1: The rotational symmetry remains exact for all possible signature-preserving (- + + + ) deformations of the sphere.

THEOREM 3.5.1: The Mandelbrocki isospaces are directly universal, in the sense of admitting as particular cases all possible spaces with the same signature \(+ + + + = 0 \), such as the Mandelbrocki, Riemannian, Euclidean and other spaces (universally), directly in terms of the isometry within fixed local variables (direct universality).

Theorem 3.5.1 states that, for isomers in the (3, 2)-plane, are given by

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x^2 = x^2 \times \cos \theta \times (g_{11} + g_{22})^{\frac{1}{2}} - x^2 \times \sin \theta \times (g_{11} + g_{22})^{\frac{1}{2}}, \tag{3.5.1a}
\]

\[
x^2 = x^2 \times \cos \theta \times (g_{11} + g_{22})^{\frac{1}{2}} + x^2 \times \sin \theta \times (g_{11} + g_{22})^{\frac{1}{2}}, \tag{3.5.1b}
\]

For the general expression in three dimensions interested reader can inspect Ref. [7] for brevity.
\[ x^2 = v^2, \quad (3.5.20a) \]
\[ x^2 = x^2 \cos^2 (\pi \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}}) - \]
\[ \left( x^2 - \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}} \right) \sin^2 (\pi \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}}) = 0, \quad (3.5.20b) \]
\[ x^2 = \left( x^2 - \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}} \right) \sin^2 (\pi \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}}) + \]
\[ \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}} \cos^2 (\pi \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}}) = y \times \left( x^2 - y^2 \right), \quad (3.5.20c) \]
\[ x^2 = \left( x^2 - y^2 \right) \sin^2 (\pi \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}}) + \]
\[ \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}} \cos^2 (\pi \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}}) = y \times \left( x^2 - y^2 \right), \quad (3.5.20d) \]
where
\[ y_z = x_z \times \frac{g_{\mu} + g_{\nu}}{g_{\mu} + g_{\nu}} \times x_z = \frac{1}{\left( g_{\mu} + g_{\nu} \right)^2}. \quad (3.5.21) \]

For the general expression interested readers can inspect Ref. \[ 7 \].

Contrary to another popular belief throughout the 20-th century, Ref. \[ 58 \] proves that

**Lemma 5.5.5**: The Lorentz symmetry remains exact for all possible signatures preserving \( c \) as such in the deformed forms of the Minkowski space.

Again, the symmetry remains exact under the use of the appropriate mathematics. The above reconstruction of the exact Lorentz symmetry can be geometrically visualized by noting that the light cone
\[ x^2 + y^2 = \left( x^2 - y^2 \right)^2 = 0, \quad (3.5.22) \]
can only be formulated in vacuum, while within physical media we have the light cone
\[ x^2 + y^2 = \frac{x^2 + y^2}{g_{\mu} + g_{\nu}} = 0, \quad (3.5.23) \]
that, when formulated on isospace over isofield, is also a perfect cone, as it is the case for the isospheres. This property then explains how the Lorentz symmetry is reconstructed as exact according to Lemma 5.5.2 or, equivalently, that (3.4.3) is isomorphic to (5.11).

(5) The isorelativizations \[ 20 \]
\[ T_{I}(\xi) = T(\xi, \xi), \quad x = \xi + 4k, \quad k = \pm 1, \pm 2, \ldots, \] that can be written
\[ x^2 = x^2 + A(\xi, \xi), \quad (3.5.25a) \]
is the isorelativity and its isodual.

**Theorem 3.5.5**: The Poincaré-Santilli isometry, herein denoted with
\[ P(13) = \{ \pi \rightarrow \Pi, \Pi \rightarrow \pi \}, \quad (3.5.28) \]
and, therefore, the conventional Poincaré symmetry, are eleven dimensional.

The increase of dimensionality of the fundamental spacetime symmetry as, predictably, for reading amidships, including a basically novel and axiomatically consistent grand unification of electromagnetic and gravitational interactions studied in Chapter 5.

The simplest possible realization of the above formalism for isorelativistic kinematics can be outlined as follows. The first application of isorelativity is that of providing an invariant description of locally varying speeds of light propagating within physical media. For this purpose a realization of isorelativity requires the knowledge of the density of the medium in which motion occurs.

The simplest possible realization of the fourth component of the isometric is then given by the function
\[ g_{\mu} = x_{\mu} (x, \ldots), \quad (3.5.29) \]
normalized to the value \( g_{\mu} = 1 \) for the vacuum (note that the density of the medium in which motion occurs cannot be specified by special relativity). The above representation then follows with invariance under \( P(\xi, \xi) \).

In this case the quantities \( x_{\mu}, \mu = 1, 2, 3 \), represent the indiscernability and isotropy of the medium considered. For instance, if the medium is homogeneous and isotropic (such as water), all metric elements coincide, in which case
\[ I = \text{Diag} (g_{\mu}, g_{\nu}, g_{\rho}) = \left[ g_{\mu} \right] \times \text{Diag} (1, 1, 1), \quad (3.5.30a) \]
\[ J = \left[ g_{\mu} \right] \times \text{Diag} (1, 1, 1), \quad (3.5.30b) \]
thus confirming that inertioes are hidden in the Minkowski sense, and this may be a reason why they were not been discovered until recently.

Next, isorelativity has been constructed for the invariant description of systems of extended, nonspatial and deformable particles under Hamiltonian and non-Hamiltonian interactions.

Practical applications then require the knowledge of the actual shape of the particles considered, here assumed for simplicity as being spherical ellipsoids with semiaxes \( a, b, c \).

Note that the minimum number of constituents of a closed non-Hamiltonian system is two. In this case we have shapes represented with \( n_{ab}, n_{ac}, n_{bc} = 1, 2, \ldots, n \).

Specific applications finally require the identification of the number of the particles, e.g., whether occurring on an extended surface or volume. As an illustration, two spinning particles denoted 1 and 2 in condition of deep mutual penetration and overlapping of their wavepackets (as it is the case for valence bonds), can be described by the following Hamiltonian and total isometric
\[ H = \frac{p_{x_1} \cdot p_{x_2}}{m_1 + m_2} + \frac{p_{x_2} \cdot p_{x_1}}{m_1 + m_2} + V(x), \quad (3.5.31a) \]
\[ I_{12} = \text{Diag} \left[ a_1^2, b_1^2, c_1^2 \right] + \text{Diag} \left[ a_2^2, b_2^2, c_2^2 \right] \times \text{Diag} \left[ a_1^2, b_1^2, c_1^2 \right] + \text{Diag} \left[ a_2^2, b_2^2, c_2^2 \right] \times \]

where \( V \) is a positive constant.

The above realization of the isometric has permitted the first known invariant and numerically exact representation of the binding energy and other features of the hydrogen, water and other molecules \[ 71,72 \] (see Chapter 9) for which is historical 25 has been missing for about one century. The above isometric has also been instrumental for a number of additional data on twobody systems whose representation had been impossible with quantum mechanics, such as the origin of the spin 1 of the ground state of the deuteron that, according to quantum axioms, should be zero.

Note in isometric \( 3.5.31 \) the noninvariance in the wave functions, the nonlocal-integral character and the impossibility of representing any of the above features with a Hamiltonian.

From the above examples interested readers can then represent any other closed non-Hamiltonian systems.

### 3.5.4 Isorelativity and Its IsoDual

The third important part of the new isorelativity is given by the following isonets of conventional relativistic axions that, for the case of motion along the third axis, can be written \[ 29 \] as follows \[ 60 \].

**Isoaxiom 1**: The projection in our spacetime of the maximal causal invariant space is given by
\[ V_{\text{iso}} = c_n^2, \quad c_n = \sqrt{n} \quad (3.5.32) \]
This axiom resolves the inconsistencies of special relativity recalled earlier for particles and electromagnetic waves propagating within physical media such as water.

In fact, water is homogenous and isotropic, thus requiring that
\[ g_{\mu} = g_{\mu} = g_{\mu} = g_{\mu} = 1/c^2, \quad (3.5.33) \]
where \( n \) is the index of refraction.

In this case the maximal causal speed for a massive particle is \( c_n \), as experimentally established, e.g., for electrons, while the local speed of electromagnetic waves is \( c_n / n \), as also experimentally established.

Note that such a resolution requires the abandonment of the speed of light as the maximal causal speed for motion within physical media, and its replacement with the maximal causal speed of particles.
It happens that in vacuum these two maximal causal speeds coincide. However, even in vacuum the correct maximal causal speed remains that of particles and not that of light, as generally believed. At any rate, physical media are generally opaque to light but not to particles. Therefore, the assumption of the speed of light as the maximal causal speed within media in which light cannot propagate would be evidently vacuous.

It is an instructive exercise for interested readers to prove that

**LEMMA 3.1.4.** The maximal causal isoupd of particles on isoinvariant space over an isofield remains $c$. 

In fact, on isospaces over isofields $c^2$ is deformed by the index of refraction into the form $c^2/c_n^2$, but the corresponding unit $c_n^2/na^2$ is deformed by the inverse amount, $c_n^2/na^2$, thus preserving the numerical value $c^2$, due to the structure of the isoinvariant studied earlier. 

The understanding of isorelativity requires the knowledge that, when formulated on the Minkowski-Santilli isospace over the isocausal, Isosymmetry I coincides with the conventional axioms that is, the maximal causal speed returns to be $c$.

The same happens for all remaining axioms.

**ISOSYMMETRY II.** The projection in our spacetime of the isorelativity addition of isospaces within physical media is given by:

$$v_{\text{media}} = \frac{\gamma_1 v_1 + \gamma_2 v_2}{1 + \frac{\gamma_1 v_1 \gamma_2 v_2}{c_n^2}} = \frac{\gamma_1 v_1 + \gamma_2 v_2}{1 + \frac{\gamma_1 \gamma_2 v_1 v_2}{c_n^2}} = c_{\text{media}}. \quad (3.5.34)$$

We have again the correct result that the sum of two maximal causal speeds in order:

$$v_{\text{total}} = c_{\text{total}} = c. \quad (3.5.35)$$

yields the maximal causal speed in water, as the reader is encouraged to verify.

Note that such a result is impossible for special relativity. Note also that the "relative" sum of two speeds of lights in water, $\gamma = c/c_n$, does not yield the speed of light in water, thus confirming that the speed of light within physical media, assuming that they are transparent to light, is not the fundamental maximal causal speed.

**ISOSYMMETRY III.** The projection in our spacetime of the isorelativity laws of dilatation of time $t_j$ and construction of length $l_i$ and the nature of mass $m_j$ with speed are given respectively by:

$$t = \gamma t_j, \quad l = l_j. \quad (3.5.36)$$

Among various applications, Isosymmetry I removes any need for the "missing mass" in the universe. This is due to the fact that all isotopic life of experimental data agree on values $g_{\text{iso}} > 1$ within the hyperdense media in the interiors of hadronic nuclei and stars [5].

As a result, Isosymmetry V yields a value of the total energy of the universe dramatically larger than that believed until now under the assumption of the universal validity of the speed of light in vacuum.

For other intriguing applications of Isosymmetries V, e.g., for the rest energy of hadronic constituents, we refer the interested reader to monographs [5], [6].

The causal isorelativity for the characterization of antimatter can be easily constructed via the isospace map of Chapter 2, and its explicit study is left to the interested reader for leisure.

### 3.5.5 Isorelativistic Hadronic Mechanics and its Isoduals

The isorelativistic extension of relativistic hadronic mechanics is readily permitted by the Poincaré-Santilli isosymmetry. In fact, the invariant (3.1.1a) characterizes the following iso-Galilean equations on $\mathbb{C}$ over $\mathbb{C}$ [55]:

$$p_{\text{iso}} \cdot \sigma = -i \mathcal{A}_{\text{iso}} \cdot \sigma - c \cdot j_{\text{iso}} \cdot \sigma. \quad (3.5.36)$$

The linearization of the above second-order equations into the Dirac-Santilli isosymmetry has been first studied in Refs. [60], [62] and then by other authors (although generally without the use of isomathematics, thus losing the invariance).

By recalling the structure of Dirac's equation as the Kronecker product of a spin 1/2 massive particle and its antiparticle of Chapter 2, the Dirac-Santilli isosymmetry is formulated on the total isoinvariant isospace and related isosymmetry

$$S^2 \sigma = \frac{S_4^2 \sigma(x, \sigma, q, \beta) = S \sigma^2(x)}{2}$$

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$$\ell = \gamma \ell_j, \quad x = \gamma x_j. \quad (3.5.36)$$

$$\beta = \frac{\gamma \beta_j + \beta \gamma_j}{\sqrt{1 - \beta_j \beta}}, \quad (3.5.4)$$

where one should note that, since the speed of light is always smaller than the maximal possible speed, $\gamma$ cannot assume imaginary values.

Note that in water these values coincide with the relativistic ones as should be once particles such as the electrons have in water the maximal causal speed $c$. Note again the necessity of avoiding the interpretation of the local speed of light as the maximal local causal speed. Note also that the mass diverges at the maximal local causal speed, but not at the local speed of light.

**ISOSYMMETRY IV.** The projection in our spacetime of the iso-Doppler law is given by the code (here formulated for simplicity for WC angle of aberration):

$$\omega = \gamma \omega_j = \omega. \quad (3.5.17)$$

This isorelativistic axiom permits an exact, numerical and unvarying representation of the large differences in cosmological redshifts between quasars and galaxies when physically connected.

In the case light simply exits the huge quasar chromospheres already redshifted due to the decrease of the speed of light, while the speed of the quasars can remain the same as that of the associated galaxy. Note again as this result is impossible for special relativity.

Isosymmetry IV also permits a numerical interpretation of the internal blue- and redshift of quasars due to the dependence of the local speed of light on its frequency.

Finally, Isosymmetry IV predicts that a component of the phenomenon toward the red of sunlight at sunset is of iso-Doppler nature. This prediction is based on the different travel within atmospheres of light at sunset compared to the noon (evidently because of the travel within a comparatively denser atmosphere).

By contrast, the popular representation of the apparent redshift of sunlight at sunset is that via the scattering of light among the molecules composing our atmosphere. Had this interpretation be correct, the sky at the zenith should be red, while it is blue.

At any rate, the claim of representation of the apparent redshift via the scattered light of light is political because of the impossibility of reaching the needed numerical value of the redshift, as serious scholars are suggested to verify.

$$\frac{d \ell}{d \theta} = \ell \frac{d \theta}{d \ell} \quad \frac{d \omega}{d \theta} = \omega \frac{d \theta}{d \omega} \quad \frac{d \beta}{d \theta} = \beta \frac{d \theta}{d \beta}$$

as the local speed of light is increased by a factor $\gamma$.

The mutation of spin then characterizes a necessary mutation of the intrinsic space $S_{\text{iso}}(x, \sigma, q, \beta)$ of Eq. (3.5.3).

Finally, Isosymmetry IV also permits a numerical interpretation of the internal blue- and redshift of quasars due to the dependence of the local speed of light on its frequency.

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as the local speed of light is increased by a factor $\gamma$.
of the shape of a charged and spinning body implies the necessary alteration of its magnetic moment.

The construction of the isomodular elastohydrodynamic mechanics is left to the interested reader by keeping in mind that the iso-Dirac equation is isomorphic as the conventional equation.

To properly understand the above results, one should keep in mind that the resolution of the isometric characteristics of particles is totally referred to the constituents of a hadronic bound state under conditions of mutual penetration of their mass units (such as one hadronic constituent) under the conditions of recovering conventional characteristics for the hadronic bound state as a whole (the hadron considered) much along Newtonian singularity constrains on non-Riemannian forces, Eq. (1.16).

It should be also stressed that the above indicated relations validate the unified condition often formulated for conventional Hilbert spaces, such consequential catastrophic incompatibilities, Theorem 1.5.2.

As an illustration, the violation of causality and probability law has been established for all eigenvalues of the angular momentum $M$ different than the quantum spectrum $M^2 = |p| = (l + 1)(l + 2)$, $l = 0, 1, 2, 3, \ldots$ (3.45)

As a matter of fact, these incompatibilities are the very reason why the mutations of internal characteristics of particles for bound states at short distances could not be admitted within the framework of quantum mechanics.

By comparison, hadronic mechanics has been constructed to recover unitarity on iso-Hilbert spaces over fields, thus permitting an invariant description of internal mutations of the characteristics of the constituents of hadronic bound states, while recovering conventional features for states as a whole.

Far from being mere mathematical curiosities, the above mutations permit basically new structure models of hadrons, nuclei, and stars, with consequential, new clean energies and fuels (see Chapters 11, 12).

These new advances are prohibited by quantum mechanics precisely because of the preservation of the isometric characteristics of the constituents in the transition from bound states at large mutual distance, for which no mutation is possible, to the bound state of the same constituents in condition of mutual penetration, in which case mutations have to be admitted in order to avoid the replacement of a scientific process with unsubstantiated personal beliefs one way or the other (see Chapter 12 for details).

3.5.6 Isogravity and its isoumlaut

As indicated in Section 1.4, there is no doubt that the classical and operator formulations of gravitation on a curved space have been the most controversial.

i) All conventional-field equations, such as the Einstein–Rothbert and other field equations, can be formulated via the Minkowsk–Santilli isoequation since the latter preserves all the tools of the conventional Riemannian geometry, such as the Christoffel’s symbol, covariant derivative, etc., [35].

ii) isogravity is isometric at the classical level and isounitary at the operator level, thus resolving the catastrophic incompatibilities of Theorems 1.5.1 and 1.5.2.

iii) An isometrically consistent operator version of gravity always existed and merely crept in unnoticed through the 20-th century because gravity is imbedded of the bending of light that is twice the experimental value, one for curvature of a Newtonian, one for newtonian attraction, this illustrating again that isogravity possesses, at the abstract level, the same inner characteristics of particles for bound states at short distances as a whole

b) The converse on the fact that gravity on a Riemannian space admits a well-defined “Euclidean”, but not “Minkowskian” limit, which controversy is trivially resolved by isogravity via the limit

$$I_{grav}(x) = 1$$

(3.5.50)

c) The resolution of the controversy on the fact that Einstein’s gravitation predicts a value of the bending of light that is twice the experimental value, one for curvature of a Newtonian, one for newtonian attraction, this controversy is trivially resolved by the elimination of curvature as the origin of the bending, as necessary in any case for the free fall of a body along a straight radial line in which no curvature of any type is conceivable possible or credible; and other controversies.

A resolution of the controversies on quantum gravity can be seen from the property that relativistic hadronic mechanics of the preceding section is a quantum formulation of gravity whenever $F = T_{grav}$.

This illustrates again that all conventional results of gravitation, including experimental verifications, can be reformulated in invariant form via isogravity.

Moreover, the problematics of general relativity mentioned earlier refer to the isometric gravitational problem. Perhaps greater problematic aspects exist in gravitation on a Riemannian space for interer gravitational problems, e.g., because of the lack of characterization of basic features, such as the density of the interior problem, the locally varying speed of light, etc.

These additional problematic aspects are also resolved by isogravity due to the unmodified character of the fundamental dependence of the isometric that,
The above lines constitute only the initial aspects of isogravitation since its most important branch is interior isogravitation as characterized by isounit and isotopic elements of the illustrative type
\[
\left( \frac{\partial^2}{\partial r^2} \right) = 1 \left( \frac{\partial^2}{\partial r^2} \right) > 0, \quad (3.5.5a)
\]
and
\[
\left( \frac{\partial}{\partial r} \right) \left( \frac{\partial^2}{\partial r^2} \right) > 0, \quad (3.5.5b)
\]
permitting a geometric representation directly on the isometric of the actual shape of the body considered, on the above case an ellipsoid with semiaxes \(a_1^2, a_2^2, a_3^2\), as well as the (average) interior density \(\bar{\rho}\) with consequential representation of the (average value of the) interior speed of light \(C = c/\bar{\rho}\).

A most important point is that the invariance of interior isogravitation under the Poincaré-Santilli isosymmetry persists in its totality since the latter symmetry is completely independent from the explicit value of the isounit or isotopic element, and solely depends on their positive-definite character.

Needless to say, isounit (3.4.53) is merely illustrative because a more accurate interior isounit has a much more complex functional dependence with a locally varying density, light-speed and other characteristics as they occur in reality.

Explicit forms of these more adequate models depend on the astrophysical body considered, e.g., whether gaseous, solid or a mixture of both, and their study is left to the interested reader.

It should also be noted that gravitational singularities should be solely referred to interior models because interior descriptions of type (3.5.52) are not necessarily engendered by the corresponding interior formulations. Consequently, the current views on black holes could well result to be pseudo-scientific beliefs because the only realistic statement that can be professed at this time without raising issues of scientific ethics is that the gravitational features of large and hyperdense aggregations of matter, whether characterizing a "black" or "brown" hole, are basically unresolved at this time.

Needless to say, exterior isogravitation is a particular case of the interior formulation. Consequently, from now on, unless otherwise specified isogravitation will be referred to the interior form.

The cosmological implications are also intriguing and will be studied in Chapter 6. It should be indicated that numerous formulations of gravitation in flat Minkowski space exist in the literature, such as Ref. 79 and papers quoted therein. However, these formulations have no connection with isometry since the background space of the former is conventional, while that of the latter is a geometric unification of the Minkowskian and Riemannian spaces.

The main structural component of Lie’s theory is its universal enveloping associative algebra \(\mathcal{Z}(L)\) of a Lie algebra \(L\). In fact, Lie algebras can be obtained as the attached antisymmetric part \(\mathcal{Z}(L)\) of \(L\); the infinite-dimensional basis of \(\mathcal{Z}(L)\) permit the exponentiation to a finite transformation group \(G\), and the representation theory is crucially dependent on the right and/or left modular associative action originally defined on \(G\).

In Section 3.2.9B we have reviewed the rudiments of the universal enveloping associative algebras \(\mathcal{Z}(L)\) of a Lie-Santilli isalgebra \(L\). It is easy to see that all features occurring for \(\mathcal{Z}(L)\) carry over to the covering isomorph \(\mathcal{G}(L)\).30

In this appendix we would like to outline a more technical definition of universal enveloping associative algebras since they are at the foundations of the unification of simple Lie algebras of dimension \(N\) into a single Lie-Santilli isalgebra of the same dimension (Section 3.2.13).

With reference to Figure 27, the envelop \(\mathcal{Z}(L)\) of a Lie algebra \(L\) can be defined as the \((\zeta, \tau)\) where \(\zeta\) is an associative algebra and \(\tau\) is a homomorphism of \(L\) into the antisymmetric algebra \(\mathcal{C}\) attached to \(\zeta\) such that: if \(\mathcal{H}\) is another associative algebra and \(\tau^0\) is another homomorphism of \(\mathcal{H}\) into \(\mathcal{C}\), a unique isomorphism \(\gamma\) exists between \(\zeta\) and \(\mathcal{H}\) in such a way that the diagram in the l.h.s of Figure 27 is commutative.

The above definition evidently expresses the uniqueness of the Lie algebras \(L\) up to local isomorphism, and illustrates the origin of the name “universal” enveloping algebra of \(L\).

With reference to the r.h.s. diagram of Figure 27, the universal enveloping associative algebra \(\mathcal{Z}(L)\) of a Lie algebra \(L\) was introduced in Ref. 4 as the set \((\xi, \tau)\) where \((\xi, \tau)\) is a conventional envelope of \(L\); \(\zeta\) is an isotopic mapping \(\xi \rightarrow \xi' \rightarrow L \rightarrow L\); \(\mathcal{C}\) is an associative algebra generally nonisomorphic to \(\xi\); \(\tau^0\) is a homomorphism of \(\mathcal{H}\) into \(\mathcal{C}\); such that if \(\mathcal{H}\) is another associative algebra and \(\tau^0\) is another homomorphism of \(\mathcal{H}\) into \(\mathcal{C}\), there exists a unique

\[\text{isomorphism } \gamma \text{ of } \xi \text{ into } \mathcal{C} \text{ with } \tau^0 = \gamma(\tau) \text{ and two unique isomorphs } (\xi') = \xi \text{ and } (\xi'^0) = \mathcal{C}.
\]

A primary objective of the above definition of isomorphism is the lack of uniqueness of the Lie algebra characterized by the isomorphism or, equivalently, the characterization of a family of generally nonisomorphic Lie algebras via the use of only one basis. The above definition of isomorphism also explains in more details the variety of realization of the simple 3-dimensional Lie-Santilli isalgebra \(L_3\) provided in Eq. (3.2.230), and may be of assistance in extending the same classification to other isalgebras.

The above notion of isomorphism represents the essential mathematical structure of hadronic mechanics, namely, the preservation of the conventional basis, i.e., the set of observables of quantum mechanics, and the generalization of the operations on them via an infinite number of isomorphisms so as to admit a new class of interactions structurally beyond the possibilities of quantum mechanics.
Appendix 3.B
Recent Advances in the TSSFN Isotopology

In Section 3.2.7 we introduced the elements of the Tsung-Szeuh-Schulli-Far-Con-Netz isotopology (or TSSFN isotopology for short). In this appendix we outline recent advances on the isotopology by the Spanish mathematicians R. M. Falco Guisemuria and J. Núñez Valdés [24,25].

**PROPOSITION 3.2.B1:** Consider a mathematical structure $(E, +, \cdot, \ldots)$.

If we construct an isotopic lifting such that:

a) Both pairs $(+, \cdot)$ and isounits $\cdot$ in the same type as in the initial, which is endowed with planes $S, I, \ldots$, with respect to $\cdot$, $\cdot$, respectively.

b) $(E, +, \cdot, \ldots)$ is a structure of the same type as the initial, which is endowed with planes $S, I, \ldots$, with respect to $\cdot$, $\cdot$, respectively.

c) $I$ is an unit with respect to $\cdot$ in the corresponding general set $\mathbb{V}$, being $\mathbb{V} = \mathbb{V}^2 \in V$ the associated isotopic element.

Then, by defining in the isotopic level the operations:

$$\mathbb{II} = (\mathbb{I}, \mathbb{S}) = \mathbb{I} \times \mathbb{S} \times \mathbb{I} = \mathbb{I} \times \mathbb{S} \times \mathbb{I} \times \mathbb{S} \times \mathbb{I} \times \mathbb{S} = \mathbb{I} \times \mathbb{S} \times \mathbb{I} \times \mathbb{S} \times \mathbb{I} \times \mathbb{S}$$

And being defined in the projection level:

$$\mathbb{II} = (\mathbb{I}, \mathbb{S}) = (\mathbb{I} \times \mathbb{S}) \times (\mathbb{I} \times \mathbb{S}) \times (\mathbb{I} \times \mathbb{S}) = (\mathbb{I} \times \mathbb{S}) \times (\mathbb{I} \times \mathbb{S}) \times (\mathbb{I} \times \mathbb{S})$$

It is obtained that the isounits $(\mathbb{I}, \mathbb{S})$ is of the same type as the initial one.

The study in Refs. [24,25] is made by taking into consideration both isotopic and projection levels. Equivalent results related to injective isotopies are also obtained. In the first place, Proposition 3.2.A1 is verified for topological spaces and for their elements and basic properties: isotopology, isoclosed sets, isopen sets, $T_0$, etc.

**PROPOSITION 3.2.B5:** They are verified that:

a) $f$ is isounitaneous if and only if the mapping $f$ from which comes from is continuous. That result is similar in the projection level by using injective isotopies.

b) Every isounitaneous is isounitaneous.

c) Isounitaneity is preserved by both topological composition and product.

Finally, the analysis of (iso)(pseudo)metric isounitances is also concreted:

**PROPOSITION 3.2.B6:** Let $\mathbb{M}$ be a $K$ isounitaneous, isotopic lifting of a vector space $\mathbb{M}$, endowed with a (pseudo)metric defined on an ordered field $\mathbb{K}$ by using an isounit which preserves the inverse element and compatible with respect to the addition in $\mathbb{M}$. Then, the isoununit $d$ is an isoununitaneous.

Let $(\mathbb{M}, d')$ be an (iso)(pseudo)metric $K$ isounitaneous, endowed with an isounit $d$ such that $d' \subseteq d$ and $d' \subseteq d$. If $\mathbb{M}$ is endowed with a (pseudo)metric $d$, then $\mathbb{M}$ is isounitaneous in $d'$.

**PROPOSITION 3.2.B7:** Under conditions of Proposition 3.2.B6, if $B_k(\mathbb{X}_0, r)$ is a metric ball in $\mathbb{M}$, then $B_k(\mathbb{X}_0, r)$ is a metric ball in $\mathbb{M}$.

A metric neighborhood of an isounit $X \subseteq \mathbb{M}$ is a subset $X \subseteq \mathbb{M}$ containing a metric ball centered in $X$. The set of metric neighborhoods of $X$ is denoted by $B^X$. Finally, if $d'$ is the iso-Euclidean isounitaneous over $\mathbb{P}^r$, the associated metric neighborhoods are called iso-Euclidean neighborhoods.

**PROPOSITION 3.2.B8:** Let $\mathbb{M}$ and $\mathbb{N}$ be two (iso)(pseudo)metrics over an isounitaneous $\mathbb{M}$. It is verified that $B^M_k(\mathbb{X}_0, r) \subseteq B^M_k(\mathbb{X}_0, r)$. If only if every metric ball $B^M_k(\mathbb{X}, r)$ contains a ball $B^M_k(\mathbb{X}, r)$ and every ball $B^M_k(\mathbb{X}, r)$ contains a ball $B^M_k(\mathbb{X}, r)$.

A topological isounit is every isounit endowed with a topological space structure. If, besides, such an isounit is an isotopic projection of a topological space, it is called isounitaneous isounit.

Similarly, there are obtained concepts of (iso)boundary isounit, closure of an isounit, closed set, isounitaneous isounit, interior of an isounit, open set, (iso)Hausdorff isounit and second countable isounit, among others.

**PROPOSITION 3.2.B10:** The space from which any topological isounit in the isotopic level is obtained can be endowed with the final topology relative to the mapping $I$.

The isotopic projection of a topological space is an isounitaneous isounit in the projection level. If such a projection is injective, then every isounitaneous isounit is such a level in, in fact, isounitaneous.

Similar results are obtained for the concepts of (iso)boundary isounit, isointersection isounit and isounitaneous isounit, among others.

**PROPOSITION 3.2.B11:** The isounits $\mathbb{M}$ and $\mathbb{N}$ are defined as orders of $\mathbb{M}$ and $\mathbb{N}$, of the same type as $\mathbb{M}$.

Let $\mathbb{U}$ be a $\mathbb{N}$ isounitaneous space with isounit $\mathbb{I}$ and isounit $\mathbb{S}$, obtained from an isounit compatible with respect to each one of the initial operations. It will be said that an isounit isounitaneous $\mathbb{I} \subseteq \mathbb{U}$ in $\mathbb{U}$, if for all $\mathbb{X} \subseteq \mathbb{U}$, there exists $\mathbb{S} \subseteq \mathbb{U}$ such that for all $\mathbb{X} \subseteq \mathbb{U}$ with $[\mathbb{X}, \mathbb{X}] \subseteq \mathbb{I}$, it is verified that $[\mathbb{I}, \mathbb{I}] \subseteq \mathbb{I}$. We will say that $\mathbb{I}$ isuncontinous in $\mathbb{U}$ if it is uncontinous in $\mathbb{X}$, for all $\mathbb{X} \subseteq \mathbb{U}$. Finally, when dealing with injective isotopies, the isounitaneity in the projection level is defined in a similar way.

**PROPOSITION 3.2.B12:** The isounitaneity in $\mathbb{U}$ is equivalent to the continuity in $\mathbb{U}$. In the case of injective isotopies, both once are equivalent to the one in $\mathbb{U}$.

The isounitaneity on isounitaneous isounit is also analyzed:

**PROPOSITION 3.2.B9:** Every isounit endowed with an (iso)(pseudo)metric is an isounitaneous isounit.

**PROPOSITION 3.2.B10:** Let $\mathbb{I} = (\mathbb{M}, d) = (\mathbb{N}, d')$ be an isounitaneous between $\mathbb{K}$ isounitaneous endowed with (iso)(pseudo)metric and let us consider $\mathbb{X} \subseteq \mathbb{M}$. Then, $\mathbb{I}$ is isounitaneous in $\mathbb{X}$ if and only if for all $\mathbb{X} \subseteq \mathbb{U}$ there exists $\mathbb{S} \subseteq \mathbb{K}$ such that $\mathbb{I} \subseteq \mathbb{S}$, and if $\mathbb{S} \subseteq \mathbb{K}$, then it is verified that $[\mathbb{S}, \mathbb{S}] \subseteq \mathbb{S}$.

**PROPOSITION 3.2.B11:** Let $\mathbb{I} = (\mathbb{M}, d) = (\mathbb{N}, d')$ be an isounitaneous between two isounitaneous $\mathbb{M}$ and $\mathbb{N}$. If conditions of the definition of isounitaneity are satisfied, then $\mathbb{I}$ is isounitaneous if and only if $\mathbb{I} = (\mathbb{U})$ is an isounitaneous.

For all isounits $\mathbb{U}$ of $\mathbb{N}$.
Appendix 3.C
Recent Advances on the Lie-Santilli Isotheory

In Section 3.2.2 we have outlined the fundamentals of the Lie-Santilli isotherapy. It may be useful for the mathematically oriented reader to outline recent developments achieved by the Spanish mathematicians E. M. Falcón Ganfornina and J. Núñez Valdés [24,25,43] in the field beyond those presented in monographs [26,36,37]. Falcón and Núñez introduced in 2001 [37] a new construction model of isotopies which was similar to the one proposed by Santilli in 1978 although in its motivated version presented by the same author later on [8] (see Chapter 4) became based on the use of several isoradicals and isosolvability operations existing in the initial mathematical structure. Such a model, which from now on will be called MCM (modified construction model based on the multiplication), was later generalized in Refs. [24,25,43]. In a schematic way, Santilli's isosolvability can be described with the following diagram:

\[
\begin{array}{c|c}
\text{Conventional Level} & \text{General Level} \\
\hline
(\mathbb{E}, +, \cdot, \ldots) & (\mathbb{E}, \bullet, \cdot, \ldots) \\
\hline
\text{Projective Level} & \text{Isosolvability} \\
(\mathbb{E}, +, \cdot, \ldots) & (\mathbb{E}, \oplus, \cdot, \ldots) \\
\end{array}
\]

where, by construction:

- The mapping \( (\mathbb{E}, +, \cdot, \ldots) \rightarrow (\mathbb{E}, \oplus, \cdot, \ldots) \) is an isomorphism.
- The isoradial projection of a Lie-Santilli isoalgebra, which constitutes a generalization of the conventional case, since they are not constants in general, but functions depending of the factors of \( \mathbb{T} \).
- Another interesting isotopy in the Santilli's Lie-admissible algebras [4], is the isoradical \( \mathbb{U} \) such that with the commutator bracket \([\cdot,\cdot]:\mathbb{X} \cdot \mathbb{Y} = (\mathbb{X} \cdot \mathbb{Y}) - (\mathbb{Y} \cdot \mathbb{X})\) is an isomorphic Lie isoalgebra. The following result is satisfied:

**PROPOSITION 3.2.C6:** Under conditions of Proposition XXX, let \( \mathbb{U} \) be a Lie algebra and let \( \mathbb{U} \rightarrow \mathbb{U} \) be a Lie isosolvable isoalgebra, then \( \mathbb{U} \) is a Lie isosolvable isoalgebra.

In this way, Santilli’s Lie-admissible isoalgebras inherit the usual properties of conventional (admissible) Lie algebras. In the same way, usual structures related with such algebras have also their analogue ones when isotopies are used. Remarkably, an isosolvable Lie isoalgebra \( \mathbb{U} \) is every isotopic lifting of an ideal \( \mathbb{I} \) of \( \mathbb{U} \); which is by itself an ideal. In particular, the center of a Lie isoalgebra \( \mathbb{U} \), \( \mathbb{X} \in \mathbb{U} \) such that \( [\mathbb{X}, \mathbb{I}] = \mathbb{X} \cdot \mathbb{I} \), is an isodal \( \mathbb{U} \). In fact, it is verified that the following result:

**PROPOSITION 3.2.C1:** Let \( \mathbb{U} \) be a Lie isoalgebra associated with a Lie algebra \( \mathbb{U} \) and let \( \mathbb{J} \) be an ideal of \( \mathbb{U} \). Then, the corresponding isotopic lifting \( \mathbb{U} \) is an isoidal \( \mathbb{U} \).

An isoidal \( \mathbb{J} \) of a Lie isoalgebra \( \mathbb{U} \), \( \mathbb{J} \subseteq \mathbb{I} \), is called isosolvable if \( \mathbb{U} \cdot \mathbb{I} = \mathbb{S} \) for all \( \mathbb{X} \in \mathbb{J} \) and for all \( \mathbb{Y} \in \mathbb{U} \), being \( \mathbb{U} \) isosolvable if it is so as its isoidal.

**PROPOSITION 3.2.C5:** \( \mathbb{U} \) is isosolvable if and only \( \mathbb{U} \) is commutative.

A Lie-Santilli isoalgebra can also be introduced as follows. Given a Lie-isoassociative isoalgebra \( \mathbb{U} \), the commutator in \( \mathbb{U} \) associated with \( \mathbb{I} \) is defined by \( [\mathbb{X}, \mathbb{Y}] = (\mathbb{X} \cdot \mathbb{Y}) - (\mathbb{Y} \cdot \mathbb{X}) \), for all \( \mathbb{X}, \mathbb{Y} \in \mathbb{U} \), one isomorphic Lie-Santilli isoalgebra associated with \( \mathbb{I} \) is a Lie-Santilli isoalgebra associated with \( \mathbb{I} \) under conditions of Proposition XXX. Then, the Lie-Santilli isoalgebra associated with \( \mathbb{I} \) is a Lie-Santilli isoalgebra under conditions of Proposition XXX.

Apart from that, a Lie-Santilli isoalgebra \( \mathbb{U} \) is said to be isosolvable if, being an isoidal of a simple Lie isoalgebra, it is not isosolvable and the only isoidal which contains is trivial. In an analogous way, \( \mathbb{U} \) is called isosolvable if, being an isoidal of a simple Lie isoalgebra, it does not contain trivial isosolvable isoids. Note that, this definition implies that every isosolvable Lie isoalgebra is also isosolvable. Moreover, it is verified that:

**PROPOSITION 3.2.C7:** Under conditions of Proposition XXX, the isotoxic lifting of a (semi)simple Lie isoalgebra is an (semi)simple Lie isoalgebra. Particularly, every isosolvable Lie isoalgebra is a direct sum of isosolvable Lie isoalgebras.
A Lie-Santilli isalgebra \( (\hat{U} \oplus \bullet) \) is called isonilpotent if, being an isonilpotent Lie algebra, in the series
\[
\hat{U} = \hat{U}^1 \supset \hat{U}^2 \supset \hat{U}^3 \supset \cdots = \hat{U}^0 = \{0\}
\]
(which is called isonilpotent series), there exists a natural integer \( n \) such that \( \hat{U}^n = \{0\} \). The number of such integers is denominated normalized index of the isalgebra.

As an immediate consequence of this definition it is deduced that every isonilpotent Lie isalgebra is isosolvable and that every nonsolvability iscommutative Lie isalgebra has an isonilpotency index 2, being 1 the corresponding of the isalgebra \( \{0\} \). Moreover, they are verified.

**PROPOSITION 3.3.C.11**: Under conditions of Proposition XXX, the isotopic lifting of a Lie isalgebra \( U \) is an isonilpotent isotopic isalgebra.

**PROPOSITION 3.3.C.12**: Let \( \hat{U} \) be a Lie isalgebra associated with a Lie algebra \( U \). They are verified:
1) Under conditions of Proposition XXX, the sum of a finite number of isonilpotent isalgebras of \( \hat{U} \) is another isonilpotent isalgebra.
2) If \( \hat{U} \) is also isonilpotent and \( \hat{U} \) is isalgebra, then
   (a) Every isonilpotent isoideal of \( \hat{U} \) is isonilpotent.
   (b) Under conditions of Proposition XXX, \( \hat{U} \) is isonilpotent, then its center is a real nil.

In a similar way as the case isosolvable, the result (1) involves that the sum of all isonilpotent isalgebras of \( \hat{U} \) is another isonilpotent isalgebra, which is denoted by useful radical of \( \hat{U} \), to be distinguished from the nil-radical of \( \hat{U} \), which is the sum of the radicals ideals. It will be represented by \( \text{nn-rad}(\hat{U}) \), which allows to distinguish it from the nil-radical \( \hat{U} \). It is immediate that \( \text{nn-rad}(\hat{U}) \subset \text{nil-rad}(\hat{U}) \subset \hat{U} \).

Apart from that, it is possible to relate an isonilpotent isotopic Lie isalgebra with its derived Lie isalgebra, by using the following:

**PROPOSITION 3.3.C.13**: Under conditions of Proposition XXX, a Lie isotopic isalgebra is isonilpotent if and only if its derived Lie isalgebra is isonilpotent.

---

**Appendix 3.D**

Lorentz versus Galileo-Roman Relativistic Symmetry

As indicated in Section 3.5.1, special relativity has remained unsettled after one century of studies, even in the arena of its original conception, namely, point-particles and electromagnetic waves propagating in vacuum. A reason of the ongoing debates is connected to the alternative of Lorentz invariance for the two-way light experiments conducted to date, and the Galilean invariances expected for one-way light experiments. The alternative of Lorentzian vs Galilean treatments is obscured by the fact that the former applies for relativistic apodes while the latter is not perceived as such. This limitation was resolved in the early 1970s by the relativistic formulation of the Galilean symmetry and relativity proposed by P. Roman, J. J. Agliardi and R. M. Santilli [76,78], and here called Galileo-Roman symmetry and relativity.1)

In short, the alternative as to whether the ultimate relativity is of Lorentzian or Galilean type is far from being resolved. It is an easy prediction that such an alternative will not be resolved in these volumes. Consequently, in this appendix we can merely review the main ideas of the Galileo-Roman symmetry, and leave the resolution of the alternative to future generations.

By assuming an in-depth knowledge of the Galileo symmetry and its scalar extension (that we cannot possibly review here), the Galileo-Roman symmetry is based on the following assumptions:

1) The carrier space is given by the Kronecker product of the conventional Minkowski space \( M(n,\mathbb{R}) \) times a one-dimensional space \( S(u) \) where \( u \) represents the proper time normalized to the dimension of length for reason clarified below,

\[
S(u) = M(1, \mathbb{R}) \times U(u)
\]

---

1) The name of "Galileo-Roman symmetry and relativity" is suggested because all basic concepts were originated by Paul Roman. In Refs. [76-78] and other papers, the author merely assisted Paul Roman in the technical elaboration of his views.
to my best recollection, we could find no experimental data contradicting the Galilean-Romano symmetry.

Yet, the novelty of the symmetry caused a real opposition from among colleagues, namely, a reaction that has to be distinguished from proper scientific scrutiny. Part of the opposition was due to the political attachment to Einsteinian doctrines, but part was also due to the fact that the Galilean-Romano group required technical knowledge above the average of the intellectual planes of the time.

Then, at the end of 1974, the author was forced to file a lawsuit against Italian physicists and their backers. Despite all the above, negative judgments are wrong unless expressed with due exceptions.

<table>
<thead>
<tr>
<th>Publications by R. M. Santilli as of 1974</th>
</tr>
</thead>
<tbody>
<tr>
<td>R. M. Santilli, Studies on coalitions, Contributed part to the International Conference on Relativistic Physics, Brussels, 1969-70</td>
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</table>

Formal development, the formal study of the noncommutative subgroups of the Galilean-Romano group, and the use of the Galilean-Romano group for the solution of the famous Einstein's field equations.
Such hope an opposition essentially forced the author to abandon the studies in the field, a decision that he regretted later, but could not change at that time due to the need in the 1970s for the author to secure an academic position so as to feed and shelter two children in tender age and his wife.

During the 37 years that have passed since that time, the author discovered numerous theories published in the best technical journals that, in reality, did verify the Galileo-Roman symmetry, but were published as verifying the conventional Poincaré symmetry. All attempts by the author for editorial corrections turned out to be useless. That was unfortunate for the fully deserved continuation of Paul Roman’s name in science.

In this way, the author was exposed for the academic rage caused by novelty and, in so doing, he acquired the necessary strength to resist academic disruptions when he proposed the construction of hadronic mechanics in 1978 [4]. In this way, the human experience gained by the author during his studies of the Galileo-Roman symmetry and relativity proved to be crucial for the proposal and continuation of the studies on hadronic mechanics against hardly credible obstructions, oppositions and disruptions.

Yet, the author hopes that studies on the Galileo-Roman symmetry and relativity are made continued by new generation of physcists, not only because of the dramatic richness of content compared to the Poincaré sub-symmetry, but also because the Galileo-Roman symmetry and the easily derivable isotopic extension appear to possess the necessary ingredients for a solution of the numerous unresolved problems of special relativity, including compatibility with the ultimate frontier of knowledge: space.
Chapter 4

LIE-ADMISSIBLE BRANCH OF HADRINOCH MECHANICS AND ITS ISOUDA

NOTE: THIS CHAPTER MUST BE COMPLETED AND EDITED

4.1 INTRODUCTION

4.1.1 The Scientific Imbalance Caused by Irreversibility

As recalled in Chapter 1, physical, chemical or biological systems are called irreversible when their images under time reversal \( t \rightarrow -t \) are prohibited by causality and/or other laws, as it is generally the case for nuclear transmutations, chemical reactions and organism growth.

Systems are called reversible when their time reversed images are as causal as the original ones, as it is the case for planetary and atomic structures when considered isolated from the rest of the universe, the structure of crystals, and other structures (see reprint volumes [1]) on irreversibility and vast literature quoted therein).

Another large scientific imbalance of the 20th century studied in these monographs is the treatment of irreversible systems via the mathematical and physical formulations developed for reversible systems, since these formulations are themselves reversible, thus resulting in serious limitations in virtually all branches of science.

The problem is compounded by the fact that all used formulations are of Hamiltonian type, under the awareness that all known Hamiltonians are reversible over time (since all known potentials, such as the Coulomb potential \( V(r) \), etc., are reversible).

The scientific imbalance was generally dismissed in the 20th century with unambiguously statements, such as “irreversibility is a macroscopic occurrence that disappears when all bodies are reduced to their elementary constituents.”
4.1.2 The Forgotten Legacy of Newton, Lagrange and Hamilton

The scientific imbalance on irreversibility was created in the early part of the 20th century when, to achieve compatibility with quantum mechanics and special relativity, the entire universe was reduced to potential forces. Formally, the analytic equations were “transcended” with the removal of the external terms.

In reality, Newton [2] did not propose his celebrated equations restricted to force derivable from a potential \( F = \partial \Phi / \partial \theta \), but proposed them for the most general possible forces,

\[
m_{a} \frac{d}{dt} F_{a}(t,r,\theta) = 1 \quad k = 1,2,3; \quad a = 1,2,3,4, \ldots, N,
\]

where the conventional associative product of numbers, matrices, operators, etc. is continued to be denoted for the symbol \( \times \) as to distinguish it from numerous other products needed later on.

Similarly, to be compatible with Newton’s equations, Lagrange [3] and Hamilton [4] decomposed Newton’s force into a potential and a nonpotential component they represented all possible forces with functions today known as the Lagrangian and the Hamiltonian, and proposed their celebrated equations with external terms,

\[
\frac{d}{dt} \{ \alpha(t,r,\theta) \} = \{ H(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1a)

\[
\frac{d}{dt} \{ H(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1b)

\[
\frac{d}{dt} F_{a}(t,r,\theta) = m_{a} \frac{d}{dt} \{ F_{a}(t,r,\theta) \} + F_{a}(t,r,\theta),
\]

(4.1c)

\[
L = \sum_{a} \frac{1}{2} m_{a} \{ v(t,r,\theta) \}^{2} - V(t,r,\theta),
\]

(4.1d)

\[
H = \sum_{a} \frac{1}{2} \{ p(t,r,\theta) \}^{2} + V(t,r,\theta),
\]

(4.1e)

\[
\{ H(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1f)

Consequently, the true Lagrange and Hamilton equations can be more technically written,

\[
\frac{d}{dt} \{ H(t,r,\theta) \} = \{ F_{a}(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1g)

4.1.3 Early Representations of Irreversible Systems

As reviewed in Section 1.5.2, the brackets of the time evolution of an observable \( \{ \alpha(t,r,\theta) \} \) in phase space according to the analytic equations with external terms,

\[
\frac{d}{dt} \{ \alpha(t,r,\theta) \} = \{ F_{a}(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1h)

\[
\frac{d}{dt} \{ H(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1i)

\[
L = \sum_{a} \frac{1}{2} m_{a} \{ v(t,r,\theta) \}^{2} - V(t,r,\theta),
\]

(4.1j)

\[
H = \sum_{a} \frac{1}{2} \{ p(t,r,\theta) \}^{2} + V(t,r,\theta),
\]

(4.1k)

\[
\{ H(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1l)

Consequently time reversal is realized via the use of Heaviside conjunction.

4.1.3 Early Representations of Irreversible Systems

As reviewed in Section 1.5.2, the brackets of the time evolution of an observable \( \{ \alpha(t,r,\theta) \} \) in phase space according to the analytic equations with external terms,

\[
\frac{d}{dt} \{ \alpha(t,r,\theta) \} = \{ F_{a}(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1m)

\[
\frac{d}{dt} \{ H(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1n)

\[
L = \sum_{a} \frac{1}{2} m_{a} \{ v(t,r,\theta) \}^{2} - V(t,r,\theta),
\]

(4.1o)

\[
H = \sum_{a} \frac{1}{2} \{ p(t,r,\theta) \}^{2} + V(t,r,\theta),
\]

(4.1p)

\[
\{ H(t,r,\theta) \} = F_{a}(t,r,\theta),
\]

(4.1q)

Consequently time reversal is realized via the use of Heaviside conjunction.
The application of nonunitary transformations to brackets (4.1.12) then led to the proposal in memoir [335] of 1978 of the following Lie-admissible operator generalization of Heisenberg's equations in their infinitesimal form

\[ i \times \frac{dH}{dt} = (H \times P) + (P \times H) \times Q + A = [A, H^b], \]  

with finite counterpart

\[ \mathcal{A}(t) = e^{-itQ} \times \mathcal{A}(0) = e^{-itQ} \times A, \]  

under the subsidiary conditions needed for consistency, as we shall see,

\[ P = Q^b. \]  

where \( P, Q \) and \( P \pm Q \) are now nonsingular operators (or matrices), and Eq. (4.1.14) is a basic consistency condition explained later in this section. Eqs. (4.1.18) (4.1.19) are the fundamental equations of hadronic mechanics. Their basic brackets are manifestly Lie-admissible and Jordan admissible with structure

\[ \{A, B\} = (A \times P) + (P \times B) - (B \times Q) + Q = (A \times T + B - B \times T + A) = (A \times R + B + B \times R + A) \]  

\[ \mathcal{T} = P = Q, \quad R = Q - P. \]  

As indicated in Section 1.5.2, it is easy to see that the application of a nonunitary transform to the parametric brackets of Eqs. (4.1.14) leads precisely to the operator brackets of Eqs. (4.1.17).

\[ U \times (x \times A \times B - q \times B + s \times A) = U \times A \times B - B \times Q + A, \]  

\[ U \times U^b \neq I, \quad P = \frac{1}{2}(U \times U^b)^{-1}. \]  

In particular, the application of any (nonunitary) nonunitary transforms preserves the Lie-admissible and Jordan-admissible character. Consequently, fundamental equations (4.1.18), (4.1.19) are “directly universal” in the sense of Lemma 1.5.2.

However, the above equations are not invariant under their own nonunitary time evolution (4.1.14) because Eqs. (4.1.11) are manifestly nonunitary.

\[ \mathcal{T} = \mathcal{T}^b = \mathcal{T}^c. \]  

Therefore, our new Lie-admissible mathematics is today known as Santilli's genomatrices, where the prefix "geno" suggested in the original proposal [335] is used in the Greek meaning of "inducing" new axioms (as compared to the prefix "iso" of the preceding chapter denoting the preservation of the axioms).

The basic idea is to lift the isomorphisms of the preceding chapter into a form that is still nowhere singular, but non-diffusion, thus implying the existence of two different generalized units, today called Santilli genounits for the description of matter, that are generally written [336]

\[ I^a = 1/T^a, \quad \hat{I}^a = 1/\hat{T}^a, \]  

\[ I^a = \mathcal{I}^a, \quad \hat{I}^a = I^a \hat{I}^a, \]  

with two additional covalent genounits for the description of antimatter [336]

\[ (I^a)^b - (I^a)^c = -(I^b)^a - (I^c)^a, \quad (\hat{I}^a)^b - (\hat{I}^a)^c = -(\hat{I}^b)^a - (\hat{I}^c)^a. \]  

Jointly, all conventional and/or isotopic products \( A \times B \) among genounits (numbers, vectors, fields, operators, etc.) are admitted as the genounits as the correct left and right units at all levels, i.e.,

\[ A \times B = A \times B^b \times B, \quad A \times B = A \times B^b, \]  

\[ A^c = A^c \times B^c, \quad A^c = A^c \times B. \]  

For all elements \( A, B \) of the set considered.

As we shall see in Section 4.3, the above basic assumptions permit the representation of irreversibility with the most primitive possible quantities, the basic units and related products.

In particular, as we shall see in Section 4.3 and 4.4, genounits permit an invariant representation of the external forces in Lagrange's and Hamilton's

\[ A \times B = A \times B^b \times B, \quad A \times B = A \times B^b, \]  

\[ A^c = A^c \times B^c, \quad A^c = A^c \times B. \]  

For all elements \( A, B \) of the set considered.
equations (4.12). As such, genonumbers are generally dependent on time, coordinates, momenta, wavefunctions and other needed variables, e.g., $\Theta = \tilde{\Theta}^2$, $\tilde{\Theta}^3$, $\tilde{\Theta}^4$, $\tilde{\Theta}^5$, $\ldots$.

In fact, the assumption of all ordered products to the right represents matter systems moving forward in time, the assumption of all ordered products to the left represents matter systems moving backward in time, with the irreversibility being represented at most by the inequality $A > B$, $A \neq B$. Similar representation of irreversible antimatter systems occurs via isonumbers.

4.2.2 Genonumbers, Genonumeral Analysis and Their Isojodals

Genonumeral systems begin to reach maturity with the discovery made, apparently for the first time in paper [13] of 1993, that the axioms of a field still hold under the ordering of all products to the right or, independently, to the left. This unexpected property permitted the formulation of new numbers, which can be best introduced as a generalization of the isonumbers [15], though they can also be independently presented as follows:

**DEFINITION 4.2.1 [15]:** Let $F = \{0, +, \cdot, x\}$ be a field of characteristic zero as per Definitions 2.1.1 and 3.2.1. Santilli's forward genofields are rings $F^+ = \{\tilde{F}(\tilde{a}, \tilde{b}, \tilde{c}, \ldots)\}$ with elements

$$\tilde{a} = \tilde{a} + \tilde{a}^\prime,$$

(4.24)

where $a \in F$, $\tilde{F} = 1/\tilde{F}$ is a non singular non-Hermitian quantity (number, matrix or operator) generally outside $F$ and $x$ is the ordinary product of $F$; the genonum $\tilde{a}$ consists with the ordinary one $a$,

$$\tilde{a} \cdot \tilde{b} = \tilde{a} + \tilde{a}^\prime, \quad \tilde{a} \cdot \tilde{b} = \tilde{a} + \tilde{a}^\prime, \quad \tilde{a} \cdot \tilde{b} \in \tilde{F},$$

(4.25)

consequently, the additive forward genonum $\tilde{b} \cdot \tilde{a} \in \tilde{F}$ commutes with the ordinary $0 \in F$, and the forward genoproduct $\tilde{a} \cdot \tilde{b}$ is such that $\tilde{F}$ is the right and left invariant of $\tilde{F}$,

$$\tilde{F} \cdot \tilde{a} = \tilde{a} \cdot \tilde{F} = \tilde{a} + \tilde{\bar{a}}^\prime, \quad \tilde{a} \cdot \tilde{F} = \tilde{a} + \tilde{a}^\prime, \tilde{a} \in \tilde{F}.$$  

(4.26)

Santilli's forward genofields verify the following properties:

1) For each element $\tilde{a} \in \tilde{F}$ there is an element $\tilde{a}^{-1}$, called forward genoinverse, for which

$$\tilde{a} \cdot \tilde{a}^{-1} = \tilde{1}, \quad \tilde{1} \cdot \tilde{a} = \tilde{1}, \quad \tilde{a} \cdot \tilde{a}^\prime = \tilde{1}.$$  

(4.27)

2) The genonum is commutative

$$\tilde{a} \cdot \tilde{b} = \tilde{b} \cdot \tilde{a}, \quad \tilde{a} \cdot \tilde{b} \in \tilde{F}.$$  

(4.28)

**4.2.3 Genogeometries and Their Isojodals**

Particularly intriguing are the genogeometries [16] (see also monographs [18] for detailed treatments). They are best characterized by a simple geometry of the isogeometries, although they can be independently defined.

As an illustration, the Minkovski-Santilli forward genospace $\tilde{M}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{R})$ over the general $R^3$ is characterized by the following spacetime, genoconformal, genometric and genoquantum

$$\tilde{x} = \tilde{x} + \tilde{x}^\prime = \tilde{x} + \tilde{x}^\prime = \tilde{x} + \tilde{x}^\prime = (\tilde{x} + \tilde{x}^\prime + \tilde{x}^\prime) \times \tilde{F},$$

(4.188)

where one should note the necessity of the multiplication by the genonum $\tilde{x}$ as a condition for the result to be in $R^3$, $\tilde{R}^3$, or $\tilde{R}^3$.

In this section we point out that "2 multiplied by 3" can be ordered to the right or to the left, and the result is not only arbitrary, but yielding different numerical results for different orderings, 2

$$\tilde{a} \cdot \tilde{b} = \tilde{a} + \tilde{a}^\prime, \quad \tilde{a} \cdot \tilde{b} = \tilde{a} + \tilde{a}^\prime, \tilde{a} \cdot \tilde{b} \in \tilde{F},$$

(4.29)

but not necessarily commutative

$$\tilde{a} \cdot \tilde{b} = \tilde{b} + \tilde{\bar{b}}^\prime,$$

(4.30)

4) The set $\tilde{F}$ is closed under the genonum

$$\tilde{a} \cdot \tilde{b} = \tilde{a} \cdot \tilde{b} = \tilde{a} + \tilde{a}^\prime, \tilde{a} \cdot \tilde{b} \in \tilde{F},$$

(4.212)

and right and left genodistributive compositions

$$\tilde{a} \cdot \tilde{b} = \tilde{a} \cdot \tilde{b} = \tilde{a} + \tilde{a}^\prime, \tilde{a} \cdot \tilde{b} \in \tilde{F},$$

(4.214)

5) The set $\tilde{F}$ verifies the right and left genodistributive law

$$\tilde{a} \cdot \tilde{b} = \tilde{a} \cdot \tilde{b} = \tilde{a} + \tilde{a}^\prime, \tilde{a} \cdot \tilde{b} \in \tilde{F},$$

(4.215)

In this way we have the forward general numbers $\tilde{F}$, the forward genonumber numbers $\tilde{F}$ and the forward genonumber numbers $\tilde{F}$; the forward genonumber notions $\tilde{F}$ can indeed be formulated but they do not constitute genofields [15]. The backward genofields and the isodual forward and backward genofields are defined accordingly. Santilli's genofields are called the first (second) kind when the genonum is (is not) an element of $F$.

The basic antiun-Poincare-Birkhoff symmetry of characteristics of genofields is illustrated by the following:

**LEMMA 4.2.1 [15]:** Genonumbers of first and second kind are fields (namely, they verify all axioms of a field).

Note that the conventional product "2 multiplied by 3" is not necessarily equal to 6 because, for isodual numbers with unit 6 it is given by $-6$ [3].
Thus recovering the fundamental complementary conditions (4.1.17) or (4.2.2).

(2) The Lie-Santilli genoalgebras characterized by the universal, jointly Lie- and Jordan-admissible brackets.

(3) The Lie-Santilli genomatrices groups

\[ G_0 \leftarrow A_0 (x^i \leftrightarrow x^j) \rightarrow A_0 (x^i \leftrightarrow x^j) = G_0 (x^i \leftrightarrow x^j) \]

Here formulated in an invariant form (see below).

4.2.5 Genosystems and Nonconservation Laws

The implications of the Santilli Lie-admissible theory are equal mathematically and physically. On mathematical grounds, the Lie-Santilli genoalgebras are "directly universal" and include as particular cases all known algebras, such as Lie, Jordan, Flexible algebras, power associative algebras, quantum, algebras, supersymmetric algebras, Kac-Moody algebras, etc. (Section 1.5).

Moreover, when computed on the genospace

\[ B^0 \equiv \langle x^i \rangle \]

of Lie's theory (see Ref. [17] for the introduction of the genosystem theory of Lie-admissible algebras on bimodules). This is due to the fact that the computation of the left action \( A \times B = A \times B = A \times B \) on \( G \) (that is, with respect to the genocuplet \( I^+ \times I^- \)) yields the same value as the computation of the conventional product \( A \times B \) on \( G \) (that is, with respect to the trivial unit 1), and the same occurs for the value of \( A \times B \) on \( C \).

The above occurrence explains the reason the structure constant and the product (2) The Lie-Santilli genoalgebras characterize by the genospace and related genounits (their isoduals), while dynamical equations for the motion backward in time of matter systems are characterized by genoproducts to the right and related genounits (their isoduals), as it is well known, the above equations are not derivable from any variational principle.

Recall also from Section 3.1 that, contrary to popular beliefs, there exist four invariants directions of time. Consequently, time reversal alone cannot rep-
The above formalism then leads to the forward genoequation for matter systems
\[ \mathbf{S} F = \mathbf{S}_t \times \mathbf{S}_x \times \mathbf{S}_z, \]
(4.3.9)
defined over the forward genosfield
\[ \mathcal{D}_F = \mathcal{D}_t^F \times \mathcal{D}_x^F \times \mathcal{D}_z^F, \]
(4.3.10)
with total forward genosfield
\[ \mathcal{E}_F = \mathcal{E}_t^F \times \mathcal{E}_x^F \times \mathcal{E}_z^F, \]
(4.3.11)
and corresponding expressions for the remaining three spaces obtained via time reversal and isoinvolute.

The logic equations are given by:
I) The forward Newton-Santilli genoequations for matter systems [14], formulated via the genodifferential calculus,
\[ \mathcal{D}_F = \mathcal{D}_t^F \times \mathcal{D}_x^F \times \mathcal{D}_z^F, \]
(4.3.12)
II) The forward isodual genoequations for antimatter systems that are characterized by the isodual map of the backward genosfields,
\[ \mathcal{D}_F^\dagger = \mathcal{D}_t^\dagger \times \mathcal{D}_x^\dagger \times \mathcal{D}_z^\dagger, \]
(4.3.13)
IV) the forward causal genoequations for automated systems characterized by time reversal of the preceding ones.

Newton-Santilli genoequations (4.3.12) are “directly universal” for the representation of all possible (real behaved) Eqs. (1.3) in the frame of the observer because they admit a multiple infinity of solution for any given nonsolvable for.

A simple representation occurs under the conditions assumed for simplicity.
\[ N = \mathcal{E}^F = \mathcal{E}^F = 1, \]
(4.3.14)
for which Eqs. (3.12) can be explicitly written
\[ \mathcal{D}_t \mathcal{E}^\dagger = \mathcal{D}_t \mathcal{E}^\dagger = m \times \mathcal{D}_F \mathcal{D}_F = \]
\[ = m \times \frac{d \mathbf{S}_t \times \mathbf{S}_x \times \mathbf{S}_z}{dt} = m \times \mathcal{D} \mathbf{S}_t \times \mathbf{S}_x \times \mathbf{S}_z = \mathcal{E}_t^F \times \mathcal{E}_x^F \times \mathcal{E}_z^F, \]
(4.3.15)
from which we obtain the genorepresentation
\[ F^{NSA} = -m \times x \times \frac{\mathcal{D} \mathbf{S}_t}{\mathcal{D} \mathbf{S}_x}, \]
(4.3.16)
that always admit solutions have left to the interested reader since in the next section we shall show a much simple, universal, algebraic solution.

As one can see, in Newton’s equations the nonpotential forces are part of the applied force, while in the Newton-Santilli genoequations nonpotential forces are represented by the genosfields, or, equivalently, by the genodifferential calculus, in a way essentially similar to the case of isotopes.

The main difference between isodual and genoequations is that isoms are Hermitian, thus implying the equivalence of forward and backward motions, while genosfields are non-Hermitian, thus implying nonreversibility.

Note also that the topology underlying Newton’s equations is the conventional, Euclidean, local-differential topology which, as such, can only represent point particles.

By contrast, the topology underlying the Newton-Santilli genoequations is given by a genotopy of the topology studied in the preceding chapter, thus permitting the representation of extended, nonspherical and deformable particles via forward genosfields, e.g., of the type
\[ \mathcal{D}_t \mathcal{E}^\dagger = \mathcal{D}_t \mathcal{E}^\dagger = m \times \mathcal{D} \mathbf{S}_t \times \mathbf{S}_x \times \mathbf{S}_z = \mathcal{E}_t^F \times \mathcal{E}_x^F \times \mathcal{E}_z^F, \]
(4.3.17)
where \( m \times k = 1, 2, 3 \) represents the semian of an ellipsoid, \( m \times k \) represents the density of the medium in which motion occurs (with more general multidimensional realizations here omitted for simplicity), and \( \mathcal{E}_t \mathcal{E}^\dagger \) constitutes a nonsymmetric matrix representing nonsymmetric forces, namely, the contact interactions among extended constituents occurring for the motion forward in time.

4.3.3 Hamilton-Santilli Genomechanics and Its Isodual

In the section we show that, once rewritten in their identical genoform (4.3.12), Newton’s equations for nonsconsistent systems are indeed derivable from a variational principle, with analytic equations possessing a Lie-admissible structure and Hamilton-Jacobi equations suitable for the first known consistent and unique operator map studied in the next section.

The most effective setting to introduce real-valued non-symmetric genosfield is in the 6N-dimensional forward genospace (genosome bundle) with local genoscoordinates and their composites
\[ \mathcal{D}_t \mathcal{E}^\dagger = \mathcal{D}_t \mathcal{E}^\dagger = m \times \mathcal{D} \mathbf{S}_t \times \mathbf{S}_x \times \mathbf{S}_z = \mathcal{E}_t^F \times \mathcal{E}_x^F \times \mathcal{E}_z^F, \]
(4.3.18)
and
\[ \mathcal{D}_x \mathcal{E}^\dagger = \mathcal{D}_x \mathcal{E}^\dagger = m \times x \times \mathcal{D} \mathbf{S}_t \times \mathbf{S}_x \times \mathbf{S}_z = \mathcal{E}_x^F \times \mathcal{E}_x^F \times \mathcal{E}_z^F, \]
(4.3.19)
where the ordinary Lie tensor and, consequently, the superspace is Lie-admissible in the sense of Albert [7].

As one can see, the important consequence of genosmechanics and its genodifferential calculus is that of turning the triple system (\( \mathcal{D} F^{NSA} \)) of Eqs. (1.5) in the bilinear form (\( \mathcal{D} F^{NSA} \)), thus characterizing a connected object in the brackets of the time evolution.

This is the central purpose for which genomechanics was built (note that the multiplicative factors represented by \( m \times k \) are fixed for each genosystem. The importance of such a formulation will be proved shortly.

It is an instructive exercise for interested readers to prove that the brackets (\( \mathcal{D} F^{NSA} \)) are Lie-admissible, although not Jordan-admissible.

It is easy to verify that the above identical formalization of Hamilton’s historical time evolution correctly reverses the time rate of variations of physical quantities in general, and of the energy in particular,
\[ \frac{\mathcal{D} E^{NSA}}{\mathcal{D} t^{NSA}} = \{A^{NSA}, H^{NSA}\} = \{A^{NSA}, H^{NSA}\} + \frac{\mathcal{D} A^{NSA}}{\mathcal{D} t^{NSA}} \]
(4.3.20a)

\[ \frac{\mathcal{D} H^{NSA}}{\mathcal{D} t^{NSA}} = \{A^{NSA}, H^{NSA}\} = \{A^{NSA}, H^{NSA}\} + \frac{\mathcal{D} A^{NSA}}{\mathcal{D} t^{NSA}} \]
(4.3.20b)

It is easy to show that genation principle (4.3.21) characterizes the following Hamilton-Jacobi-Santilli genoformulas [14]
\[ \frac{\mathcal{D} A^{NSA}}{\mathcal{D} t^{NSA}} + \frac{\mathcal{D} A^{NSA}}{\mathcal{D} t^{NSA}} = \{A^{NSA}, H^{NSA}\} = \{A^{NSA}, H^{NSA}\} + \frac{\mathcal{D} A^{NSA}}{\mathcal{D} t^{NSA}} \]
(4.3.21a)

\[ \frac{\mathcal{D} A^{NSA}}{\mathcal{D} t^{NSA}} = \{A^{NSA}, H^{NSA}\} = \{A^{NSA}, H^{NSA}\} + \frac{\mathcal{D} A^{NSA}}{\mathcal{D} t^{NSA}} \]
(4.3.21b)

which confirm the property (crucial for genorepresentation as shown below) that the genonation is indeed independent of the linear momentum.

It is easy to verify that the above identical formalization of Hamilton’s historical time evolution correctly reverses the time rate of variations of physical quantities in general, and of the energy in particular,
The latter are reformulated via geometrodynamics as the only known way to achieve invariance and derivability from a conventional principle while admitting a consistent algebra in the brackets of the time evolution [38].

Therefore, Hamilton-Santilli geonumbers (14.66) are indeed irreversibly for all possible irreversible Hamiltonians, as desired. The origin of irreversibility rests in the contact nonpotential force $\mathbf{P}_{\text{con}}$ according to Lagrange’s and Hamilton’s teaching that is merely reformulated in an invariant way.

The above Lie-admissible mechanics requires, for completeness, three additional formulations, the backward geonumbers for the description of matter moving backward in time, and the modulus of both the forward and backward geonumbers for the description of antisymmetry.

The construction of these additional mechanics is left to the interested reader for brevity.

4.4 LIE-ADMISSIBLE OPERATOR MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

4.4.1 Basic Dynamical Equations

A simple geometry of the nature or symplectic quantization applied to Eqs. (13.24) yields the Lie-admissible branch of hadronic mechanics [38] computing four different formulations, the forward and backward geonumbers for matter and their isoduals for antimatter. The forward geonumbers for matter is characterized by the following main topics.

1) The nonreversible (thus everywhere invertible) non-Hermitian forward geonumbers for the representation of all effects causing irreversibility, such as contact nonpotential interactions among extended particles, etc. (see the subsequent chapter for various realizations)

\[ \mathbf{\dot{P}} = \left(\mathbf{J}\mathbf{P}\mathbf{J}\right)^{-1}, \]

with corresponding ordered product and general $\mathbf{R}$ and genocomplex-field geo-genoformulas.

2) The forward genonic general position $\mathbf{\hat{W}}$ with forward geonumbers $(\Psi^>)$ and forward pedestrian product

\[ \langle r^< | \Psi^> | s^> = \Psi^\dagger(r^<) \chi^r s^> = s^> \mathbf{P}^\dagger \mathbf{\hat{C}}, \]

and fundamental property

\[ \mathbf{\dot{P}}^> = (\Psi^>) \mathbf{\dot{P}} \Psi^> = (\Psi^>) \mathbf{\dot{P}} > \mathbf{\dot{P}} = \mathbf{\hat{P}}, \]

holding under the condition that $\mathbf{\dot{P}}$ is indeed the correct unit for forward motion in time, and forward geonumery transversals

\[ \Psi^\dagger > \mathbf{\dot{P}} \Psi^> = (\Psi^>) \mathbf{\dot{P}} > \mathbf{\dot{P}} = \mathbf{\hat{P}}; \]

under which the geonumber values of the geonumbers recover the conventional Planck’s unit as in the isotopic case,

\[ (\mathbf{\hat{P}} > \mathbf{\dot{P}} > \mathbf{\hat{P}} > \mathbf{\dot{P}} = I). \]

The following comments are now in order. Note first in the geonumeral principle note the crucial independence of insertion $\mathbf{\hat{P}}$ in the linear momentum, as expressed by the Hamilton-Jacobi-Santilli geonumeralizations (4.3.25). Such independence assures that geonumeralization yields a geonumericaly Projectionally self-dependent on time and coordinates, $\Psi^> = \Psi^\dagger(r^<, t)$. Other geno-Hamiltonian mechanics studied previously [7] do not verify such a condition, thus implying geonumericalyzations with an explicit dependence also on linear moments, $\Psi^> = \hat{\Sigma}^\dagger(r, t, \mathbf{P})$ that violate the abstract identity of quantum and hadronic mechanics whose treatment in any case is beyond our operator knowledge at this writing.

4.4.2 Simple Construction of Lie-Admissible Theories

As it was the case for the isotopes, a simple method has been identified in Ref. [44] for the construction of Lie-admissible (geo-)theories from any given conventional, classical or quantum formulation. It is in identifying the geonumeral as the product of two different geonumerical transforms,

\[ \mathbf{\hat{P}} = (\mathbf{\hat{P}} > \mathbf{\dot{P}} > \mathbf{\hat{P}} > \mathbf{\dot{P}} = I, \]

and subjecting the total of quantities and their operations of conventional models to dual transformations,

\[ I \rightarrow \mathbf{\hat{P}} = I \times \mathbf{\hat{P}} > \mathbf{\dot{P}}, \quad \mathbf{\hat{P}} \times \mathbf{\hat{P}} = \mathbf{\dot{P}} \times \mathbf{\hat{P}}, \]

and subjecting the total of quantities and their operations of conventional models to dual transformations,

\[ \mathbf{\hat{P}} = \mathbf{\hat{P}} \rightarrow \mathbf{\hat{P}} \times \mathbf{\hat{P}}, \quad \mathbf{\hat{P}} \times \mathbf{\hat{P}} = \mathbf{\hat{P}} \times \mathbf{\hat{P}}, \]

and subjecting the total of quantities and their operations of conventional models to dual transformations.
As a result, any given conventional, classical or quantum model can be easily lifted into the genotopic form.

In fact, under irreversibility, the value of a nonconserved energy at a given time indeed be omitted for notational simplicity, but only after the understanding of the last equation, momentum and other quantities with their arrow of time for motion backward in past times. This explains the reason for having represented in this section energy, momentum and other quantities with their arrow of time, which confirm the property of Section 4.2, namely, that under the necessary mathematical achievement of invariance under nonunitarity and irreversibility via the use of nongenouniteries, provided that such nongenounities is applied to the totality of the formalism to avoid evident inconsistencies caused by mixing different mathematics for the selected physical problem.

Let us note that, due to decades of protracted use it is easy to predict that physicists and mathematicians may be tempted to treat the Lie-admissible branch of hadronic mechanics with conventional mathematics, whether in part or in full. Such a posture would be equivalent, for instance, to the elaboration of the spectral emission of the hydrogen atom with the genounitard calculus, resulting in an evident nonscientific setting.

An invariant was first achieved by Santilli in Ref. [32] of 1997 and can be illustrated by reformulating any given nonunitary transform in the genounitary form

\[ U = U_1 \times T^{\pm \omega}, \quad W = W_1 \times T^{\pm \omega}, \]

and then showing that amounts, groupoups, groupoposition, etc., are indeed invariant under the above genounitary transform in exactly the same way as conventional units, products, exponentiations, etc. are invariant under unitary transforms.

\[ F \rightarrow F' = U_1 \times T' \times W_1 \times T^{\pm \omega}, \]

\[ \lambda \rightarrow \lambda' = (\lambda_1) T^{\pm \omega} \]

\[ W \rightarrow W' = W_1 \times T^{\pm \omega}, \quad E \rightarrow E' = 1 + T^{\pm \omega}, \]

\[ \text{for which all remaining invariances follow, thus resolving the catastrophic inconsistencies of Theorem 1.3.} \]

\[ \text{4.4.3 Invariance of Lie-Admissible Theories} \]

Recall that a fundamental axiomatic feature of quantum mechanics is the invariance under time evolution of all numerical predictions and physical laws, which invariance is due to the unitary structure of the theory. However, quantum mechanics is reversible and can only represent in a scientifically beyond academic beliefs reversible systems verifying total conservation laws due to the asymptotic character of the brackets of the time evolution.

As indicated earlier, the representation of irreversibility and nonconservation requires theories with a nonunitary structure. However, the latter are afflicted by the catastrophic inconsistencies of Theorem 1.3.

The only resolution of such a basic impasse known to the author has been the achievement of invariance under nonunitarity and irreversibility via the use of nongenounities, provided that such nongenounities is applied to the totality of the formalism to avoid evident inconsistencies caused by mixing different mathematics for the selected physical problem.

Let us note that, due to decades of protracted use it is easy to predict that physicists and mathematicians may be tempted to treat the Lie-admissible branch of hadronic mechanics with conventional mathematics, whether in part or in full. Such a posture would be equivalent, for instance, to the elaboration of the spectral emission of the hydrogen atom with the genounital calculus, resulting in an evident nonscientific setting.

An invariant was first achieved by Santilli in Ref. [32] of 1997 and can be illustrated by reformulating any given nonunitary transform in the genounitary form

\[ U = U_1 \times T^{\pm \omega}, \quad W = W_1 \times T^{\pm \omega}, \]

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\[ F \rightarrow F' = U_1 \times T' \times W_1 \times T^{\pm \omega}, \]

\[ \lambda \rightarrow \lambda' = (\lambda_1) T^{\pm \omega} \]

\[ W \rightarrow W' = W_1 \times T^{\pm \omega}, \quad E \rightarrow E' = 1 + T^{\pm \omega}, \]

\[ \text{for which all remaining invariances follow, thus resolving the catastrophic inconsistencies of Theorem 1.3.} \]
It is easy to see that the time evolution of the Hamiltonian is given by
\[
\frac{dH}{dt} = -i[H, \mathcal{A}(t)] = -i[A, H] = -A_i \partial_i H,
\]
thus correctly reproducing behavior (5.2).

The operator image of the above dissipative system is straightforward. Physically, we are also referring to a first approximation to a massive and stable elementary particle, such as an electron, penetrating within hadronic matter (such as a medium). Being stable, the particle is not expected to "disappear" at the initiation of the dissipative force and be converted into "virtual states" due to the inability of represent such a force, but more realistically the particle is expected to experience a rapid dissipation of its kinetic energy and perhaps after that participate in conventional processes.

Alternatively, we can say that an electron relating in an atomic structure does indeed evolve in time with conserved energy, and the system is indeed Hamiltonian. By the idea that the same electron when in the core of a star also evolves that participate in conventional processes.

The operator image can be characterized by the genounits and related genoequations
\[
\mathcal{A}(t) = e^{iA t} \Lambda(t)[e^{-iA t}],
\]
and related infinitesimal form, the Heisenberg-Santilli genoequations
\[
\frac{dA}{dt} = A < H - H > = A < H^{\prime} - H^{\prime} > A,
\]
correctly represent the considered dissipative system.

By noting that the Lie-brackets in Eqs. (4.5.9) are conventional, we seek a resolution of the geno-elements for which the Lie brackets attached to the Lie-admissible brackets (5.12) are conventional and the symmetric brackets are Jordan-isotopic. A solution is then given by [32]
\[
\Gamma = 1 - \Gamma, \quad \Sigma = 1 + \Gamma,
\]
and for Eq. (5.12) becomes
\[
\frac{dA}{dt} = [\mathcal{A}(H - H^{\prime}) - (\mathcal{A}^{\prime} + \mathcal{H} A^{\prime})] = A [\mathcal{H}, A],
\]
for which the Lie-admissible brackets (5.12) are conventional.

Moreover, the "direct universality" of Lie-admissible representations requires the conventional Hamiltonian representing the physical total energy, and the genounit for forward motion in time representing the NSA forces, according to the equations
\[
H = \Sigma_0 + V(r), \quad I^\pi = \left(\begin{array}{c} 0 \\ \Sigma_{0,0} \end{array}\right),
\]
under which we have the equations of motion (for \(\mu, \nu = 1, 2, 3, \ldots, 6N\)) [32]
\[
\frac{d\Sigma^\mu}{dt} = \left(\begin{array}{c} 0 \\ \Sigma_{0,0} \end{array}\right), \quad \frac{d\Sigma^\nu}{dt} = \left(\begin{array}{c} 0 \\ \Sigma_{0,0} \end{array}\right),
\]
the classical, finite, Lie-admissible genoequations
\[
\mathcal{A}(t) = \exp\left(\frac{1}{\Sigma_{0,0}} \frac{dH}{dt} \right) \mathcal{A}(0) \exp\left(-\frac{1}{\Sigma_{0,0}} \frac{dH}{dt} \right),
\]
with infinitesimal time evolution
\[
\frac{dA}{dt} = \left(\begin{array}{c} 0 \\ \Sigma_{0,0} \end{array}\right) \frac{dH}{dt} \mathcal{A}^{\prime} - \left(\begin{array}{c} 0 \\ \Sigma_{0,0} \end{array}\right) \frac{dH}{dt} \mathcal{A}^{\prime}\mathcal{A}^{\prime} = [A, \mathcal{H}],
\]
yielding the correct nonconservation of the energy
\[
\frac{dH}{dt} = -\Sigma^\beta \frac{dH}{dt},
\]
and related Heisenberg-Santilli genoequations
\[
\frac{dA}{dt} = [A, H - H^{\prime} - (A, H^{\prime}) = \quad [A, H^{\prime}] + (A^{\prime} H)^{\prime}
\]
that correctly represent the time rate of variation of the nonconserved energy,
\[
\frac{dH}{dt} = \Sigma^\beta \frac{dH}{dt}.
\]

The uninitiated reader should be incidentally aware that generally different genouns may be requested for different generators, as identified since Ref. [11].

In the latter operator case we are referring to an extended, massive and stable particle, such as a proton, penetrating within high energy, in which case the rapid decay of the kinetic energy is caused by contact, resistive, integrals of nonlocal type, e.g., occurring over the volume of the particle.

The advantages of the Lie-admissible formulations over presently-existing representation of nonconservative systems should be pointed out. Again, a primary advantage of the Lie-admissible treatment is the characteristic of the nonconserved Hamiltonian with a formulation, thus observable quantity, a feature generally absent in other treatments.

Moreover, the "direct universality" of Lie-admissible representations requires the following comments. Recall that coordinates transformations have indeed been used from the very beginning in the coordinate systems of nonconservative systems because, under such a framework, the existence of coordinate transformations \(\left(\begin{array}{c} r \end{array}\right) \rightarrow \left(\begin{array}{c} r^d, \rho_j \end{array}\right)\) under which a system that is non-Hamiltonian in the original coordinate becomes Hamiltonian in the new coordinates (see Ref. [6] for details). However, the needed transformations are necessarily nonlinear with serious physical consequences, such as:

1) Quantities with direct physical meaning in the coordinates of the new system, such as the Hamiltonian \(H(r, p) = \Sigma + V(r)\), are transformed into quantities that, in the new coordinates, have purely mathematical meaning, such as \(H(r^d, p^d) = N X(p^d), N \in R\), thus preventing any physically meaningful operator treatment.

2) There is the loss of any meaningful experimental verifications, since it is impossible to place any measurement apparatus in mathematical coordinates such as \(r^d = \Sigma X(p^d) = \Sigma X(p^d)\).

3) There is the loss of Galileo’s and Einstein’s special relativity, trivially, because the new coordinates \(\left(\begin{array}{c} r^d \end{array}\right)\) characterize a highly noninvariant image of the original inertial system of the experiment.

All the above, and other infeasibilities are resolved by the Lie-admissible treatment of nonconservative systems.

4.5.3 Pauli-Santilli Lie-Admissible Matrices

Following the study of the nonconservation of the energy, the next important topic is to study the behavior of the conventional quantum spin under contact nonconservative forces, a topic studied for the first time in memoar [32]. For this objective, it is most convenient to use the method of Sections 4.4.2 and 4.4.3, namely, subject the conventional Pauli’s matrices to two different nonunitary transforms. To avoid unnecessary complexity, we select the following two

\[
\begin{align*}
&\left(\begin{array}{c}
\mathcal{A}(t) = e^{iA t} \Lambda(t)[e^{-iA t}]
\end{array}\right) = e^{iA t}, \quad [\mathcal{A}(t), \mathcal{A}(t)] = [A, H] = [A, H^{\prime}] = [A, H^{\prime}],
\end{align*}
\]
where \(\left[\mathcal{A}, H\right]\) are a conventional Lie brackets as desired, and \([\mathcal{A}, H^{\prime}]\) are Jordan-isotopic brackets. The desired representation then occurs for
\[
\frac{dH}{dt} = -\Sigma^\beta \frac{dH}{dt} = -\Sigma^\beta \frac{dH}{dt}.
\]
Note that the achievement of the above operator form of system (5.1) without the Lie-admissible structure would have been impossible, to our knowledge. Despite its elementary character, the above illustration has deep implications. In fact, the above example constitutes the only known operator formulation of a dissipative system in which the nonconserved energy is represented by a formulation of the operator \(H\), thus being an observable despite its nonconservative character. In all other cases existing in the literature the Hamiltonian is generally non-formulation, thus non-observable.

The latter occurrence may illustrate the reason for the absence of a consistent operator formulation of nonconservative systems throughout the 20-th century until the advent of the Lie-admissible formulations.
metrics

\[ A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & b \end{pmatrix}, \quad AA \neq I, \quad BB \neq I, \quad (4.6.1) \]

where \( a \) and \( b \) are non-null real numbers, under which we have the following forward and backward geometric and related genotypic elements

\[ J^+ = BA^+ = \begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix}, \quad J^- = BA^- = \begin{pmatrix} 1 & 0 \\ 0 & -b \end{pmatrix}. \quad (4.6.2a) \]

\[ J = BA = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}, \quad J^- = BA^- = \begin{pmatrix} 1 & 0 \\ 0 & -b \end{pmatrix}. \quad (4.6.2b) \]

The forward and backward Pauli-Santilli geometries are then given respectively

\[ \sigma_i^+ = A \sigma_i B = \begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix}, \quad \sigma_i^- = A \sigma_i B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (4.6.3a) \]

\[ \sigma_1^+ = A \sigma_1 B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1^- = A \sigma_1 B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (4.6.3b) \]

\[ \sigma_3^+ = A \sigma_3 B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_3^- = A \sigma_3 B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (4.6.3c) \]

in which the direction of time is embedded in the structure of the metrics. It is an instructive exercise for the interested reader to verify that computational commutation rules and eigenvalues of Pauli’s matrices are preserved under forward and backward genospace.

\[ \sigma_i^+ > \sigma_i^- > \sigma_i^+ > 2 \mu \sigma_i^2. \quad (4.6.4a) \]

\[ \sigma_i^+ > \sigma_i^- > \sigma_i^+ > 2 \mu \sigma_i^2. \quad (4.6.4b) \]

\[ \sigma_1^+ < \sigma_3^- < \sigma_3^- < \sigma_1^+ < \sigma_3^- < \sigma_1^+. \quad (4.6.4c) \]

We can, therefore, conclude by stating that Pauli’s matrices can indeed be lifted in such an irreversable way to represent the direction of time in their very structure. However, in solving the conventional notion of spin is lost in favor of a covering notion in which the spin becomes a locally varying quantity, as expected to a proton in the core of an star.

Consequently, the Lie-admissible formulation of Pauli matrices confirms the very title of memoir [32] proposing the construction of hadronic mechanics.

4.5.4 Minkowski-Santilli Irreversible Genospace

One of the fundamental axiomatic principles of hadronic mechanics is that irreversibility can be directly represented with the background geometry and, more specifically, with the metric of the selected geometry. This requires the necessary transition from the conventional geometric metrics used in the 20-th century to covering nonsymmetric geometries.

To show this structure, we study in this section the geometry of the conventional Minkowskian spacetime and related geometry with the conventional metric \( g = \text{diag} (1, 1, 1, -1) \) and related spacetime elements \( x^2 = x^0 x^0 = c^2, x = (x^0, x^1, x^2, x^3), \quad x^2 = c^2, \quad c = 1 \).

Therefore, we introduce the following four-dimensional non-Hermitian, nonsymmetric and real-valued forward and backward metrics

\[ F = CD = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (4.6.5) \]

where \( p \neq q \) are non-null real numbers, under which we have the following forward and backward genospace.

\[ \sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}. \quad (4.6.6) \]

resulting in the forward and backward nonsymmetric geometrics

\[ \sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}. \quad (4.6.7) \]

The above genospace includes, as particular case, an irreversible formulation of the Minkowskian geometry, where irreversibility is represented at the ultimate geometric foundations, the basic unit and the metric. It should be indicated that the above genospace is isomorphic to the Minkowskian geometry, with intriguing implications for the mathematical model known as geometric locomotion studied in detail in monograph [75] via the isometrics of the Minkowskian geometry. In fact, a main unresolved problems is the deformational direction of the geometry as needed to permit the geometric locomotion in one preferred direction of space. An inspection of the above genospace elements (4.6.7) clearly shows that the geometries are preferable over the isometrics for the geometric locomotion, as well as, more generally, for more realistic geometric characterization of irreversible processes.

The construction of the Lorentz-Santilli genospacetime is elementary, due to their formal identity with the isometric case of Chapter 3, and its explicit construction left as an instructive exercise for the interested reader.

4.5.5 Dirac-Santilli Irreversible Genoequation

To complete the illustrations in particle physics, we now outline the simplest possible genospace of Dirac’s equation via the genospaces of the preceding two sections, one for the spin content of Dirac’s equation and the other for its spacetime structure. Also, we shall use Dirac’s equation in its original nonrepartitioned representation a direct product of one electron and one positron, the latter without any need of second quantization (see monograph [73] for detail). In turn, the latter genonrepartition requires the use of the Isomorphisms \( A = \text{diag} (1, 1) \) as being distinct from Minkowiski. Under the above clarifications, the forward Dirac genoequations have referred to be written

\[ \sigma^+ \sigma^- \gamma^+ \gamma^- - \text{im} \gamma^+ \gamma^- > 0 \quad (4.6.8a) \]

\[ \gamma^+ \gamma^- \sigma^+ \sigma^- - \text{im} \gamma^+ \gamma^- > 0 \quad (4.6.8b) \]

with forward genonmatrices

\[ \gamma^+ = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \quad (4.6.8a) \]

\[ \gamma^- = \begin{pmatrix} 0 & B^* \\ B & 0 \end{pmatrix} \quad (4.6.8b) \]
4.5.6 Dunning-Davies Lie-Admissible Thermodynamics

A scientiﬁc imbalance of the 20th century has been the lack of interconnections between thermodynamics, on one side, and classical and quantum mechanics, on the other side. This is due to the fact that the very notion of entropy, indexed by a set of all thermodynamical laws, are centrally dependent on irreversibility, while classical and quantum Hamiltonian mechanics are structurally reversible (since all known potentials are reversible in time).

As recalled in Section 4.1, said lack of interconnection was justiﬁed in the 20th century on the belief that the nonconservative forces responsible for irreversibility according to Laplace and Hamilton, are “istic” in the sense that they only exist at the classical level and they “disappear” when passing to elementary particles, since the latter were believed to be completely reversible. In this way, thermodynamics itself was turned into a sort of “istic” discipline.

This imbalance has been resolved by hadronic mechanics beginning from its inception. In fact, Theorem 1.3.3 has established that, far from being “istic,” nonconservative forces originate at the ultimate level of nature, that of elementary particles in conditions of mutual penetration causing contact nonpotential (NSA) interactions. The inefﬁciency rooted in the inability by quantum mechanics to represent nonconservative forces, rather than by nature. In fact, hadronic mechanics was proposed and developed precisely to reach an operator representation of the nonconservative forces originating irreversibility along the legacy of Lagrange and Hamilton.

As a result of the efforts presented in this chapter, we now possess not only classical and operator theories, but more particularly we have a new mathematics, the genomathematics, whose basic axioms are not irreversible under time reversal emerging from the basic units, numbers and differentials.

Consequently, hadronic mechanics does indeed permit quantitative studies of the expected interplay between thermodynamics and classical as well as operator mechanics. These studies were pioneered by J. Dunning-Davies [30] who introduced the ﬁrst known study of thermodynamics via methods as structurally irreversible as their basic laws, resulting in a formulation we borrow today Dunning-Davies Lie-admissible thermodynamics. This section is dedicated to a review of Dunning-Davies studies.

Let us use conventional thermodynamical, a classical form of thermodynamics, and the simple construction of irreversible formulations via two different complex valued $A$ and $B$. Then, the ﬁrst law of thermodynamics can be lifted from its conventional formulation, that via reversible mechanics, into the form permitted by genomathematics:

$$Q = \Delta Q + \Delta Q^\ast = \Delta Q^\ast (\Delta U) + \Delta U^*, \text{ etc.}$$

from absence of operator forms, Hermitian conjugation is complex conjugation. For the second law we have

$$\Delta \Omega = TdS \rightarrow \Delta \Omega^* = dS^* \rightarrow dS^* = \Delta S^*$$

Domino effects permitted the ﬁrst known formulation of entropy with a time arrow, the only causal forms being that forward in time. When the genomath does not depend on the local variables, the above genomathen is reduced to the conventional one identically, e.g.

$$\Delta S^* = dS^* \rightarrow \Delta S^* = \Delta S^*$$

where $
abla _\Omega$ is a new mathematics, the Legacy of Lagrange and Hamilton.

5.7 Ongoing Applications to New Clean Energies

A primary objective of Volume II is to study industrial applications of hadronic mechanics to new energies that are under development at the time of writing this ﬁrst volume [2002]. Hence, we close this chapter with the following preliminary remarks.

The societal, let alone scientiﬁc implications of the proper treatment of irreversibility are rather serious. One planet is afﬂicted by increasingly catastrophic climatic events mandating the search for basically new, environmentally acceptable, eco-friendly, for which scope the studies reported in these monographs were initiated.

All known energy sources, from the combustion of carbon dating to prehistoric times to the nuclear energy, are based on irreversible processes. By comparison, all established doctrines of the 20th century, such as quantum mechanics and special relativity, are reversible, as recalled in Section 4.

It is then easy to see that the serious search for basically new energies requires basically new theories that are as structurally irreversible as the process they deal with are expected to describe. At any rate, all possible energies and fuels that could be predicted by quantum mechanics and special relativity were discovered by the middle of the 20th century. Hence, the assistance in continuing to restrict new energies to verify preferred reversible doctrines may cause a condemnation by posterity due to the environmental implications.

An effective way to illustrate the need for new irreversible theories is given by nuclear fusions. All efforts to date in the ﬁeld, whether for the “cold fusion” or the “hot fusion,” have been mainly restricted to verify quantum mechanics and special relativity. However, whether “hot” or “cold,” all fusion processes are irreversible, while quantum mechanics and special relativity are reversible.

It has been shown in Ref. [31] that the failure to date by both the “cold” and the “hot” fusion to achieve industrial value is primarily due to the treatment of irreversible nuclear fusions with reversible mathematical and physical methods.

In the event of residual doubt due to prolonged use of preferred theories, it is sufﬁcient to compute the quantum mechanical probability for two nuclei to “ fuse” into a third one, and then compute its time reverse image. In this way the serious scholar will see that special relativity and quantum mechanics may predict a fully causal spontaneous appearance of nuclei following their fusion, namely, a prediction outside the boundary of science.

The inclusion of irreversibility in quantitative studies of new energies suggests the development, already partially achieved at the industrial level (see Chapter 8 of Ref. [28]), of the new, controlled “intermediate fusion” of light nuclei [31], that is, a fusion occurring at minimal threshold energies needed: 1) To verify conservation laws; 2) To expose nuclei as a pre-requisite for their fusion (a feature absent in the “cold fusion” due to insufﬁcient energies), and 3) To prevent uncontrollable instabilities (as occurring at the very high energies of the “hot fusion”).

It is hoped that serious scholars will participate with independent studies on the irreversible treatment of new energies, as well as on numerous other open problems, because in the ﬁnal analysis lack of participation in basic advances is a gift of scientiﬁc priorities to others.
HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

References


Chapter 5

HYPERSTRUCTURAL BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

5.1 The Scientific Imbalance in Biology

In our view, the highest scientific imbalance of the 20th century has been the treatment of biological systems (herein denoting DNA, cells, organisms, etc.) via conventional mathematics, physics and chemistry because of various reasons studied in detail in Chapter 1.1.

We have limit ourselves to recall that biological events, such as the growth of an organism, are irreversible over time, while the mathematics of the 20-th century and related formulations are structurally reversible, that is, reversible for all possible Hamiltonians. Therefore, any treatment of biological systems via conventional mathematics, physics and chemistry because of various reasons studied in the preceding chapter are insufficient for in depth treatments of biological systems.

5.2 The Need in Biology of Irreversible Multi-Valued Formulations

It is possible to see that, despite their generality, the invariant irreversible grandformulations studied in the preceding chapter are insufficient for in depth treatments of biological systems.
As an example, in their current formulations, hyperstructures (see, e.g., Ref. [2]) lack a well-defined left and right unit thus lacking applicability to the measurement, they do not have convolutional operators, but rather the so-called weak operators, thus lacking applicability to experiments; they are not structurally irreducible, and they lock uncertainty. Consequently, conventional hyperstructures are not suitable for applications in biology.

5.3 Rudiments of Santilli Hyper-Mathematics and Hypermechanics

After a number of trials and errors, a yet broader mathematics verifying the above five conditions was identified by R. M. Santilli in monographs [5] of 1995 and in works [4,5], and subsequently studied by R. M. Santilli and the mathematician T. Vougiouklis in paper [8] of 1996 (see also mathematical study [7]). These studies resulted in a formulation today known as Santilli hypermathematics. For an in depth study, including the all crucial Lie-Santilli hypertheory, we refer the reader to the mathematical treatments [4,7]. By assuming an in-depth knowledge of genomathematics of the preceding chapter, we here limit ourselves to indicate that the selected hypermathematics is based on the assumption that the single-valued forward and backward genomes of the preceding chapter although replaced with the following multi-valued hyperoperators:

\[
F^f(x_1, x_2, x_3, x_4) = \text{Diag} \left[ I^f; \bar{I}^f \right] = \\
= \text{Diag} \left[ \left[ \bar{I}_1, \bar{I}_2, \bar{I}_3, \bar{I}_4 \right] \left[ I_1, I_2, I_3, I_4 \right] \right].
\]

(5.1a)

\[
F^b(x_1, x_2, x_3, x_4) = \text{Diag} \left[ \left[ \bar{I}_1, \bar{I}_2, \bar{I}_3, \bar{I}_4 \right] \left[ I_1, I_2, I_3, I_4 \right] \right],
\]

(5.1b)

with corresponding ordered hyperproducts to the right and to the left

\[
A > B = A \cdot^f B, A < B = A \cdot^b B.
\]

(5.2a)

\[
F > A = A \cdot^f F, F < A = A \cdot^b F.
\]

(5.2b)

Following the hyperlinking of the mother chapter, we reach the following basic equations of the multi-valued hyperstructural branch of hadronic mechanics, first proposed by Santilli in monographs [5] of 1995 (see also the mathematical works [4,6], here written in the finite and infinitesimal forms

\[
\frac{dA}{dt} = A \cdot^f \frac{dH}{dt} - H \cdot^b \frac{dA}{dt},
\]

(5.3a)

\[
A H(f) = \mu_1^f A^f + \mu_2^b A^b + \mu_3^f A^f A^b + \mu_4^b A^b A^f
\]

(5.3b)

quoted in footnote 15 of Chapter 1, where the multidimensional character of all quantities and their operations is assumed.

In the above expression the reader should recognize the diagonal elements of the matrices of the preceding chapter and then identify the multi-valued character for each diagonal element. Consequently, the above mathematics is not 3x3-dimensional, but rather it is 3-dimensional and multi-valued, namely, each axis in three-dimensional space can assume different values.

5.4 Rudiments of Santilli Iodual Hypermathematics

The iodual hypermathematics can be constructed via the iodual map of Chapter 2 here expressed for an arbitrary operator \( \hat{A} \):

\[
A(f) = \sum_{i=0}^{\infty} \hat{A}^f \left( \frac{\hat{a}^{(i)}}{i!} \right),
\]

(5.4a)

\[
A(f) = \sum_{i=0}^{\infty} \hat{A}^b \left( \frac{\hat{a}^{(i)}}{i!} \right),
\]

(5.4b)

and

\[
A^f = \sum_{i=0}^{\infty} \hat{A}^f \left( \frac{\hat{a}^{(i)}}{i!} \right),
\]

(5.4c)

\[
A^b = \sum_{i=0}^{\infty} \hat{A}^b \left( \frac{\hat{a}^{(i)}}{i!} \right),
\]

(5.4d)

in Chapter 2 (see also Figure 2.2) we have studied the need for different times. We now have the four different hypertimes for: 1) Motion forward to

\[
\tau^f = \tau(x^f, x^b, \hat{\xi}^a, \hat{\xi}^b)
\]

(5.5a)

and

\[
\tau^b = \tau(x^f, x^b, \hat{\xi}^a, \hat{\xi}^b)
\]

(5.5b)

in the 20-th century, such as attempting an understanding the DNA code via numbers dating back to biblical times, are manifestly insufficient.

The above features appear to be necessary for the representation of biological systems. As an example, consider the association of two atoms in a DNA producing an organism composed by a very large number of atoms, such as a liver. A quantitative treatment of this complex event is given by representing the two atoms with \( a \) and \( b \) and by representing their association in a DNA with the hyperproduct. The resulting large number of atoms \( a \cdot b \) in the organism is then represented by the ordered multi-valued character of the hyperproduct, such as

\[
\begin{align*}
\tau^f & = \tau(x^f, x^b, \hat{\xi}^a, \hat{\xi}^b) \\
\tau^b & = \tau(x^f, x^b, \hat{\xi}^a, \hat{\xi}^b)
\end{align*}
\]

(5.7)

The above attempt at descyphering the DNA code is another illustration of our view that the complexity of biological systems is simply beyond our comprehension at this time. A mathematical representation will eventually be achieved in due time. However, any attempt at its "understanding" would face the same difficulties of attempting to understand infinite-dimensional Hilbert space in quantum mechanics, only the difficulties are exponentially increased for biological structures.
future times characterized by \( \gamma^2 \). 2) Motion backward to past time characterized by \( \gamma^2 \). 3) Motion forward to future time characterized by \( \gamma^2 \). 4) Motion forward from past times characterized by \( \gamma^2 \). The main difference between the four times of Chapter 2 and the four hyperfunctions of this chapter is that the former are single-valued while the latter are multi-valued.

Note again the necessity of the isodual map to represent all four possible time evolutions. In fact, the conventional mathematics, such as that underlying special relativity, can only represent two of four possible time evolutions, motion forward to future time and motion backward to past time, the latter reached via the conventional time reversal operation.

The following intriguing and far reaching aspect emerges in biology. Until now we have strictly used isodual theories for the sole representation of isomatter. However, Bliet [5] has shown that the representation of the bifurcations in sea shells requires the use of all four directions of time.

The latter aspect is an additional illustration of the complexity of biological systems. In fact, the occurrence implies that the "innate time" of a seashell, that is, the time perceived by a sea shell as a living organism, is so complex to be beyond our comprehension at this writing. Alternatively, we can say that the complexity of hyperfunctions is intended to reflect the complexity of biological systems.

In conclusion, the achievement of isomorphic representations of biological structures and their behavior can be one of the most productive frontiers of science, with far reaching implications for other branches, including mathematics, physics and chemistry.

As an illustration, a mathematically consistent representation of the non-Newtonian propulsion of snail in all, the way up to big heights, automatically provides a model of geometric propulsion studied in Volume II, namely propulsion caused by the alteration of the local geometry without any external applied force.

5.5 Santilli Hyperrelativity and Its Isodual

All preceding formulations can be embodied into one single axiomatic structure subverted in monographs [3,5] and today known as Santilli hyperrelativity and its isodual, that are characterized by:

1) The irreversible, multi-valued, forward and backward, Minkowsko-Santilli hyperrelativistic equations of motion and forward and backward hyperdilations over forward and backward hyperfields, and their isoduals.

\[ \begin{align*}
\mathcal{M}(\gamma^2, \vec{q}, \vec{R}), \mathcal{M}^\perp = \mathcal{M}^{\perp}\mathcal{M}^{-} & = \mathcal{M}^{\perp}\mathcal{M}^{\perp} = \mathcal{M}^{\perp}\mathcal{M}^{\perp} = \mathcal{M}^{\perp}\mathcal{M}^{\perp} = \mathcal{M}^{\perp}\mathcal{M}^{\perp}.
\end{align*} \]

2) The corresponding irreversible, multi-valued, forward and backward Minkowski-Santilli hyperdilation and contraction, forward and backward hyperdilations over forward and backward hyperfields, and their isoduals:

\[ \begin{align*}
\mathcal{V}(\gamma^2, \vec{q}, \vec{R}), \mathcal{V}^\perp = \mathcal{V}^{\perp}\mathcal{V}^{-} & = \mathcal{V}^{\perp}\mathcal{V}^{\perp} = \mathcal{V}^{\perp}\mathcal{V}^{\perp} = \mathcal{V}^{\perp}\mathcal{V}^{\perp} = \mathcal{V}^{\perp}\mathcal{V}^{\perp}.
\end{align*} \]

3) The corresponding irreversible, multi-valued, forward and backward Minkowski-Santilli hyperaxiom of the preceding chapter over a multi-valued realization of the local coordinates and their operations:

\[ \begin{align*}
\mathcal{X}(\gamma^2, \vec{q}, \vec{R}), \mathcal{X}^\perp = \mathcal{X}^{\perp}\mathcal{X}^{-} & = \mathcal{X}^{\perp}\mathcal{X}^{\perp} = \mathcal{X}^{\perp}\mathcal{X}^{\perp} = \mathcal{X}^{\perp}\mathcal{X}^{\perp} = \mathcal{X}^{\perp}\mathcal{X}^{\perp}.
\end{align*} \]

A few comments are now in order:

i) Hyperrelativity and its isodual are the most general forms of relativities known at this writing that can be formulated on numbers verifying the axioms of a field, thus admitting a well defined left and right unit and consequent applicability to measurements.

ii) Hyperrelativity and its isodual are invariant under their respective time hyperfunctions, thus predicting the same mathematical results at different times and being applicable to experiments.

iii) Hyperrelativity and its isodual are multi-valued rather than multi-dimensional, namely, they permit the representation of multi-universes in a form compatible with our sensory perception of space-time.

iv) The speed of light in vacuum \( c_0 \) has been assumed to remain unchanged under hyperrelativistic effects, thus meaning that the speed of light is the same for all vacuum foliations of space-time.

v) Like all other quantities, hyperspeeds in general and, in particular, the hypervelocity of light must necessarily be multi-valued for consistency, namely, essentially given by the Poincare-Santilli genosymmetry of the preceding chapter.

The reader should recall that the Poincare symmetry is eleven-dimensional as projected by the discovery permitted by reformulation of the additional, 11th dimensional boson mediating fields (Sect. 5.4).

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\end{align*} \]
Appendix 5.A

Eric Trell's Hyperbiological Structures TO BE COMPLETED AND EDITED.

A new conception of biological systems providing a true advance over rather primitive prior conceptions, has been recently proposed by Erik Trell (see Ref. (164) and contributions quoted therein). It is based on representative blocks which appear in our space to be next to each other, thus forming a cell or an organism, while having in reality hyper-orientations, thus having the structure of hyper-numbers, hypermathematics and hyperrelativities, with consequential descriptive capacities immersed beyond those of pre-existing, generally single-valued and reversible biological models. Regrettably, we cannot review Trell's new hyperbiological model to avoid an excessive length, and refer interested readers to the original literature (164).

Postscript

In the history of science some basic advances in physics have been preceded by basic advances in mathematics, such as Newton's invention of calculus and general relativity relying on Riemannian geometry. In the case of quantum mechanics the scientific revolution presupposed the earlier invention of complex numbers. With new numbers and more powerful mathematics to its disposition, physics could be lifted to explain broader and more complex domains of physical reality.

The recent and ongoing revolution of physics, initiated by Prof. Ruggiero Maria Santilli, lifting the discipline from quantum mechanics to hadronic mechanics, is consistent with this pattern, but in a more far-reaching and radical way than earlier liftings of physics made possible from extensions of mathematics. Santilli realized at an early stage that basic advances in physics required invention of new classes of numbers and more adequate and powerful mathematics stemming from this. His efforts to develop such expansions of mathematics started already in 1967, and this enterprise went on for four decades. Its basic novelty, architecture and fruits are presented in the present volume. During this period a few dozen professional mathematicians world wide have made more or less significant contributions to fill in the new Santilli fields of mathematics, but the honor of discovering these vast new continents and work out their basic topologies is Santilli and his alone. These new fields initiated by Santilli made possible the realization of so-called Lie-admissible physics.

For this achievement Santilli in 1990 received the honor from Estonia Academy of Science of being appointed as the creator of his own new branch of mathematics. In 1995 he was also able to discover the fifth force of nature (in cooperation with Professor Animalu), the contact force inducing total overlap between the wave packets of the two touching electrons constituting the isoelectron. This proved highly successful in explaining the strong force by leaving behind the non-valence bounds. Powerful industrial-ecological technology exploiting these new hadronic technologies has already been highly successful already. Without the preceding advances in mathematics, the mere existence of this technology would not have been around. The mere existence of this technology is sufficient to demonstrate the significance of hadronic math-

implies a reconfiguration of conventional basic notions in the discipline. This is, as Kuhn noted, typically easier for younger and more emergent scientific minds. Until Santilli the number 1 was silently taken for granted as the primary unit of mathematics. However, as noted by mathematical physicist Peter Rowlands at University of Liverpool, the number 1 is already loaded with assumptions, that can be worked out from a lifted and broader mathematical framework. A partial and rough analogy might be linguistics where it is obvious that a universal science of language must be worked out from a level of abstraction that is higher than having to assume the word for mother to be the first word.

Santilli detrivialized the choice of the unit, and invented isomathematics where the ren was the lifting of the conventional multiplicative unit (i.e. conservation of its topological properties) to a matrix isounit with additional arbitrary functional dependence on other needed variables. Then the conventional unit could be described as a projection and deformation from the isounit by the link provided by the so-called isotopic element inverse of the isounit. This represented the creation of a new branch of mathematics sophisticated and flexible enough to treat systems entailing sub-systems with different units, i.e. more complex systems of nature.

Isomathematics proved necessary for the lifting of quantum mechanics to hadronic mechanics. With this new mathematics it was possible to describe extended particles and abandon the point particle simplification of quantum mechanics. This proved highly successful in explaining the strong force by leaving behind the non-linear complications involved in quantum mechanics struggle to describe the relation between the three baryon quarks in the proton. Isomathematics also provided the mathematical means to explain the neutron as a bound state of a proton and an electron as suggested by Rutherford. By means of isomathematics Santilli was also able to discover the fifth force of nature (in cooperation with Professor Animalu), the contact force inducing total overlap between the wave packets of the two touching electrons constituting the isoelectron. This was the key to understanding hadronic superconductivity which also can take place in fluids and gasses, i.e. at really high temperatures. These advances from hadronic mechanics led to a corresponding lifting of quantum chemistry to hadronic chemistry and the discovery of the new chemical species of magnesics with non-valence bonds. Powerful industrial-ecological technology exploiting these theoretical insights was invented by Santilli himself from 1989 on.

Thus, the development of hadronic mathematics by Santilli was not only motivated by making advances in mathematics per se, but also by its potential to facilitate basic advances in physics and beyond. These advances have been shown to be highly successful already. Without the preceding advances in mathematics, the new hadronic technology would not have been around. The mere existence of this technology is sufficient to demonstrate the significance of hadronic math-

References

hadronic mathematics. It is interesting to note that the directing of creative mathematics into this path was initiated by a mathematical physicist, not by a pure mathematician. In general this may indicate the particular potential for mathematical advances by relating the mathematics to unsolved basic problems in other disciplines, as well as to real life challenges.

In the history of mathematics it is not so easy to find parallels to the achievements made by Santilli, due to hadronic mathematics representing a radical and general lifting, relegating the previous mathematics to a subclass of isomathematics, in some analogy to taking the step from the Earth to the solar system. However, the universe also includes other solar systems as well as galaxies.

In addition to isonumbers, Santilli invented the new and broader class of genonumbers with the possibility of asymmetric genofields for forward vs. backward genonumbers, and designed to describe and explain irreversibility, characteristic for more complex systems of nature. Quantum mechanical approaches to biological systems never achieved appreciable success, mainly due to being restricted by a basic symmetry and hence reversibility in connected mathematical systems. It represented an outstanding achievement of theoretical biology when Chris Illert in the mid-1990s was able to find the universal algorithm for growth of sea shells by applying hadronic geometry. Such an achievement was argued not to be possible for more restricted hyperdimensional geometries as for example the Riemannian. This specialist study in conchology was the first striking illustration of the potency as well as necessity of isonumber and genonumber fields, and designed to describe and explain irreversibility, characteristic for more complex systems of nature.

Following the lifting from isomathematics to genomathematics, Santilli also established one further lifting, by inventing the new and broader class of hyperstructures or Santilli hypernumbers. Such hypernumbers are multivalued and suitable to describe and explain even more complex systems of nature than possible with genonumbers. Due to its irreversibility multidimensional structure hypermathematics seem highly promising for specialist advances in fields such as genetics, genetics and communication theory. By the lifting to hypermathematics hadronic mathematics as a whole may be interpreted as a remarkable step forward in the history of mathematics, in the sense of providing the essential and sufficiently advanced and adequate tools for mathematics to expand into disciplines such as anthropology, psychology and sociology. In this way it is possible to imagine some significant bridging between the two cultures of science: the hard and the soft disciplines, and thus amplifying a tendency already represented to some extent by complexity science.

The conventional view of natural scientists has been to regard mathematics as a convenient bag of tools to be applied for their specific purposes. Considering the architecture of hadronic mathematics, this appears more as only half of the truth or one side of the coin. Besides representing powerful new tools to study

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the general history of science, but also in the specific history of mathematics. Hadronic mathematics provided the necessary fuel for raising scientific revolutions in other hadronic sciences. This is mathematics that matters for the future of our world, and hopefully Santilli’s extraordinary contributions to mathematics will catch fire among talented and ambitious young mathematicians for further advances to be made. The present Special Issue volume ought to serve as an excellent appetizer in this regard.

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nature, hadronic mathematics also manifests with a more intimate and inherent connection to physics (and other disciplines), as well as to Nature itself. In this regard hadronic geometry may be of special interest as an illustration: hadromatic geometry provided the new notions of a supra-Euclidean insospace as well as its anti-isomorphic insospace, and the mathematics to describe projective deformations of geometrical relations from insospace and its insambient Euclidean space. However, these appear as more than mere mathematical constructs. Illert showed that the universal growth pattern of sea shells could be found only by looking for it as a trajectory in a hidden insospace, a trajectory which is projected into Euclidean space and thereby manifest as the deformed growth patterns humans observe by their senses. Further, the growth pattern of a certain class of sea shells (with bifurcations) could only be understood from the addition and recognition of four new, non-trivial time categories (predicted to be discovered by hadronic mechanics) which manifest as information jumps back and forth in Euclidean space. With regard to sea shell growth, one of this non-trivial time categories could only be explained as a projection from insambient spacetime. This result was consistent with the physics of hadronic mechanics, analysing masses at both operator and classical level from considering matter and antimatter (as well as positive and negative energy) to exist on an equal footing in our universe as a whole and hence with total mass (as well as energy and time) cancelling out as zero for the total universe. To establish a basic physical comprehension of Euclidean space constituted as a balanced combination of matter and antimat, it was required to develop new mathematics with isonumbers and integral numbers basically mirroring each other. Later, corresponding anti-isomorphies were achieved for genonumbers and hypernumbers with their respective insoundals.

Thus, there is a striking and intimate correspondence between the insambient architecture of hadronic mathematics and the insambient architecture of hadronic mechanics (as well as of hadronic chemistry and hadronic biology). Considering this, one might claim that the Santilli inventions of new number fields in mathematics represent more than mere inventions or constructs, namely discoveries and reconstructions of an ontological architecture being for real also outside the formal landscapes created by the imagination of mathematics and logic. This opens new horizons for treating profound issues in cosmology and ontology.

One might say that with the rise of hadronic mathematics the line between mathematics and other disciplines has turned more blurred or dotted. In some respect this represents a return to the Pythagorean and Platonist foundations of mathematics in the birth of western civilization. Hadronic mathematics has provided much new food for thought and further explorations for philosophers of science and mathematics.

If our civilization is to survive despite its current problems, it seems reasonable to expect Santilli to be honored in future history books not only as a giant in...